MATHEMATICS

Data Management

Distance Learning

Alberta EDUCATION

Module 6

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Mathematics 9

Module 6

Data Management

Alberta
EDUCATION

Mathematics 9
Student Module Booklet
Module 6
Data Management
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This document is intended for	
Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	

The Learning Technologies Branch has an Internet site that you may find useful. The address is as follows:

http://ednet.edc.gov.ab.ca/ltb

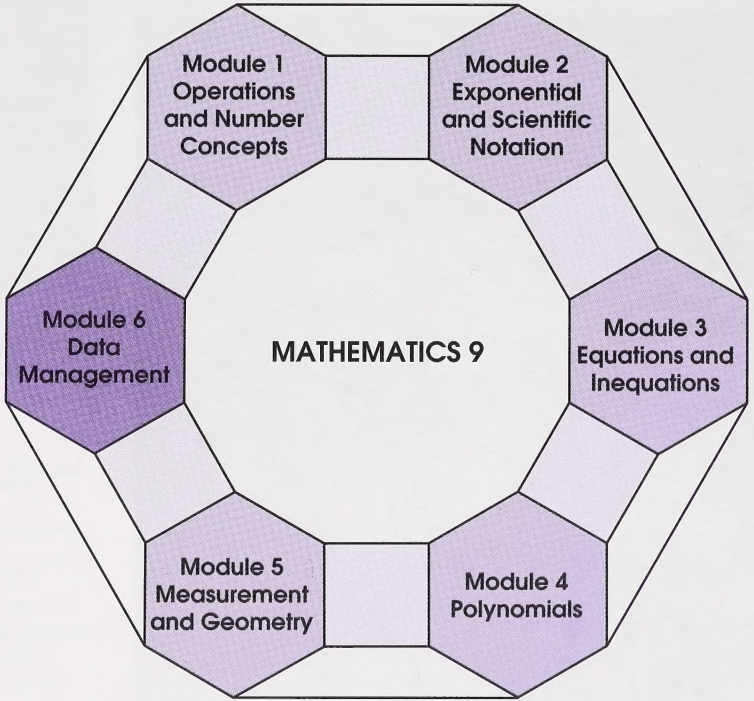
The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

Welcome

JIM WHITMER PHOTOGRAPHY

Welcome to Module 6. We hope you'll enjoy your study of Data Management.

Mathematics 9 contains six modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.



The document you are presently reading is called a Student Module Booklet. You may find visual cues or icons throughout it. Read the following explanations to discover what each icon prompts you to do.

- Prepare for a problem that will provide a change of topic.

- Prepare for a challenging problem related to the topic of the activity.

- Use the Internet to explore a topic.

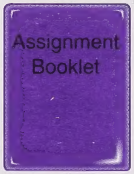
- Use computer software.

- Use a scientific calculator.

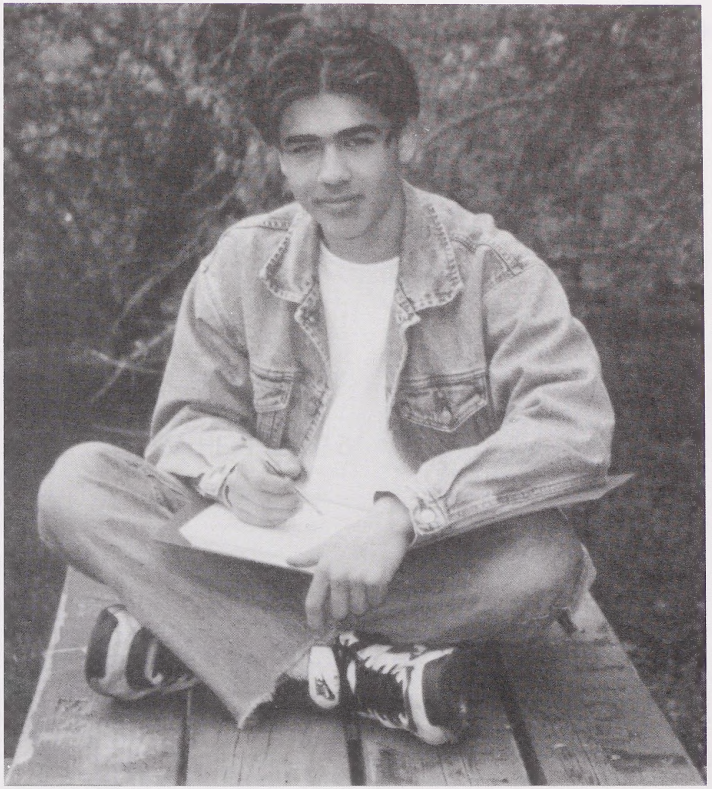
- View a videocassette.

- Pay close attention to important words or ideas.

- Use the suggested answers in the Appendix to correct activities.



- Answer the questions in the Assignment Booklet.



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There are no response spaces provided in this Student Module Booklet. This means that you will need to use your own paper for your responses. You should keep your response pages in a binder so that you can refer to them when you are reviewing or studying.

Technology

Today society is turning to **technology** more than ever before, and it is to your advantage to be able to effectively use technology when required.

Technology is the application of tools, materials, and processes to the solution of problems. More specifically, technology refers to devices and systems that are used in processing, transferring, storing, and communicating information through electronic media.

In Mathematics 9, along with the course materials, you will use a calculator, computer, and videocassette player as tools for learning and doing mathematics.

Calculators are helpful tools for solving problems and exploring patterns and relationships between numbers. Using a calculator will also save you time and help you develop your estimating skills. Therefore, you will be given numerous opportunities in each module to use a calculator.

Computers are useful for organizing and displaying data, or drawing figures. For this reason you will have the chance in many activities to work with popular computer applications such as spreadsheets and draw programs. You will also want to check out the many Internet connections in each module.

Videocassette players allow you to view video programs on key concepts that are difficult to explain in print. That is why video programs are cited in this course.

It is expected that all students will be able to view the video programs and use a calculator, and that most students will do the computer activities. However, if you are unable to access a computer, you may do the calculations using a calculator or draw figures and graphs by hand.

JIM WHITMER PHOTOGRAPHY

Problem-Solving Skills

One of the exciting features of this course is that you will develop and improve your ability in problem solving. You will need these problem-solving skills many times in your lifetime. Since this course focuses on problem solving, it is important that you understand what a **problem** is.

A problem is a task for which the method of finding the answer (as well as the answer) is not immediately known.

Like any skill, the skill of problem solving must be developed. Problems may or may not involve computation (adding, subtracting, multiplying, and dividing). Some problems are realistic; others are puzzles.

You will have the opportunity in most activities to try a problem-solving challenge.

The  icon is a cue that the problem will be related to the topic of the activity.

The  icon is a cue that the problem will provide a change of topic.

The Four-Stage Process

There are four stages that can be used to solve any problem: understanding the problem, developing a plan, trying the plan, and looking back.

Understanding the Problem

In this stage you should expect to feel puzzled. There are various reasons for feeling this way.

- You may not know the meanings of all the words.
- You may not understand the situation in the problem.
- You may be confused by unnecessary information.

Once you understand the problem, you should think about the problem and make an estimate of what the answer should be. This will help you arrive at a reasonable answer.

Developing a Plan

This is where you should decide on the plan of action that you are going to take to solve the problem. You may consider the following strategies:

- changing your point of view
- using objects
- using diagrams
- making an organized list
- using Venn diagrams
- making a table
- guessing, checking, and revising
- acting out a problem
- working backwards
- simplifying a problem
- finding and applying a pattern
- using elimination
- using truth tables
- using an equation

Note: The Appendix in this module explains these strategies in detail. When you see a problem-solving icon in any module, you should turn to this Appendix and review the problem-solving strategies.

Trying the Plan

In this stage you should try the plan and see if it works.

Be sure to work carefully and record your progress. You are encouraged to use a calculator to help with your calculations.

Note: While trying the plan, you should monitor your progress in order to determine if your plan will lead to a solution. You may find that the plan will not produce a solution, in which case a new plan will have to be developed.

Looking Back

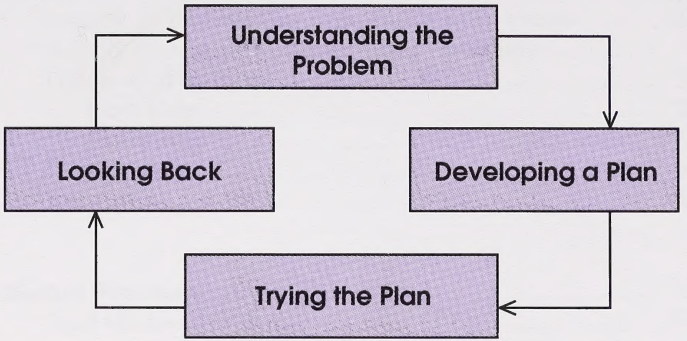
In this stage you should look back at the problem and compare your answer to the estimate you made in the first stage. Restate the problem using your answer.

Ask yourself these questions: "Did my plan work? Is my answer reasonable?"

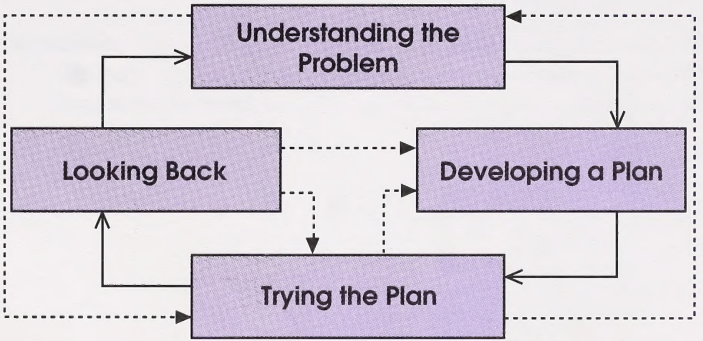
If you did not arrive at an answer, another strategy may work better. If your answer is unreasonable, you may have made errors while trying your plan.

Sequence of Stages

You usually approach a problem in the order outlined in the following diagram.



If you encounter difficulties in your original plan, or if you realize that another strategy will have better results, you may need to return to an earlier stage or use the stages in a different sequence.



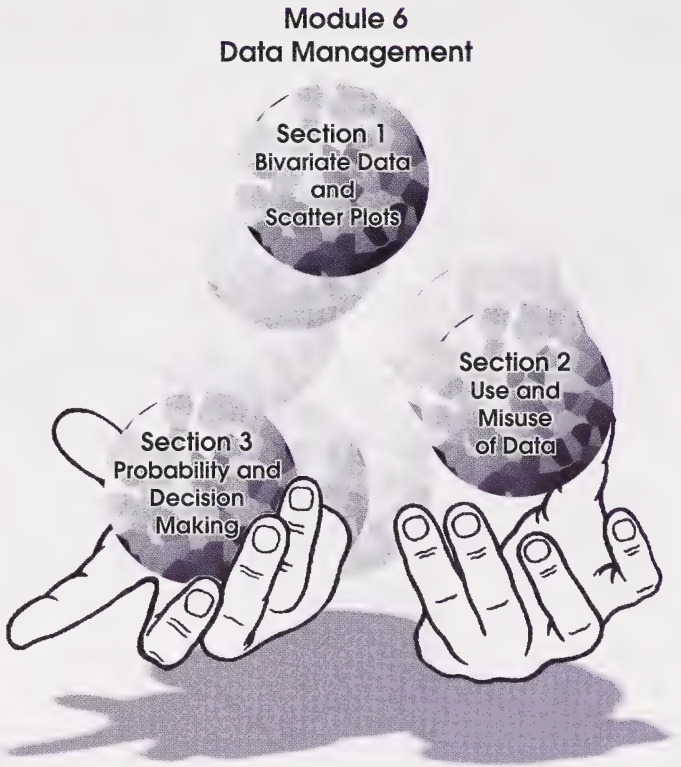
CONTENTS

Module Overview

In the modern world, you are constantly being bombarded with information. It comes at you on an ever-increasing number of channels on your television set, in daily and weekly newspapers, and in a never-ending supply of magazines that cover almost every imaginable subject. And now with the growing popularity of computers, you can gain access to a wide variety of networks covering a vast area of information through the Internet.

You certainly need to be able to properly assess and digest this information overload. A good grasp of all the tools of data management will help you in this regard.

In this module you will discover how to design experiments so that you can gather information appropriately. You will then display and interpret the data that you have gathered. Later, you will analyse how statistics is used to influence your decisions. Then, finally, you will become more knowledgeable about probability and the role it plays in analysing data and applying it to problem solving.

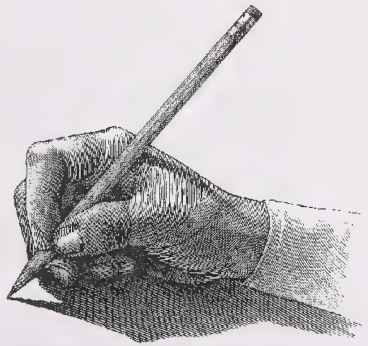


Evaluation

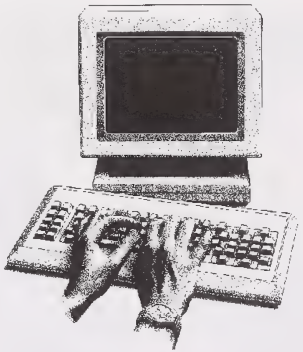
Your mark for this module will be determined by how well you complete the assignments at the end of each section and at the end of the module. In this module you must complete three section assignments and a final module assignment. The mark distribution is as follows:

Section 1 Assignment	**25 marks**
Section 2 Assignment	**15 marks**
Section 3 Assignment	**20 marks**
Final Module Assignment	**40 marks**
TOTAL	**100 marks**

When doing the assignments, work slowly and carefully. You must do each assignment independently, but if you are having difficulties, you may review the appropriate section in this module booklet.



If you are working on a CML terminal, you will have a module test as well as a module assignment.



Note

There is a supervised final test at the end of this course. Your mark for the course will be determined by how well you do on the module assignments and the supervised final test.

Section 1: Bivariate Data and Scatter Plots

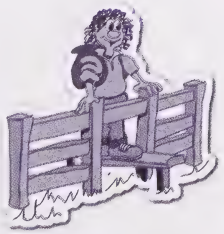
Is there any relationship between the height of an individual and his or her shoe (or skate) size? Certainly as you grow older and taller, your feet continue to grow until you are mature. But, does a tall person have larger feet than a short person? Can you predict a person's shoe size if you know his or her height or vice versa?

These and other relationships can be explored by conducting controlled surveys or by properly designing and carrying out experiments. This is how scientists, medical researchers, and people in many other occupations discover relationships.

In this section you will design experiments so you can discover the information you are seeking. Then, you will organize this data on a scatter plot and interpret what the scatter plot shows. You will become adept at drawing a line of best fit on your scatter plot and using it to make predictions.

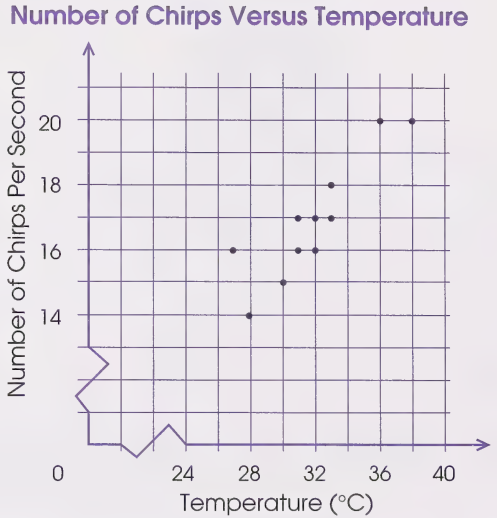
Activity 1: Examining Bivariate Data

Jolene does research for a government agency. In one area of her research, she has data on the number of cricket chirps versus temperature. One set of data is as follows.



Temperature (°C)	Number of Chirps Per Second
28	14
31	16
32	17
30	15
33	17
32	16
36	20
31	17
27	16
38	20
33	18

To graph the data, Jolene puts the number of chirps per second on the vertical axis and the temperature on the horizontal axis. The resulting graph shows the relationship between the two measures.



Up until now you have dealt with data (such as range, mean, median, and mode) that involves only one variable in your study of statistics. In this activity you will analyse data that involves a relationship between two measures. This type of data is known as **bivariate data**.

Other examples of bivariate data are as follows:

- age group and number of traffic violations
- jogging and length of life
- amount of television watched and school marks

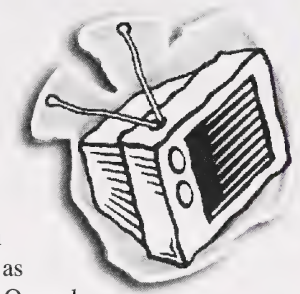
1. State three examples of bivariate data other than those listed previously.

Check your answer by turning to the Appendix.

You can design and conduct experiments that illustrate the relationships of bivariate data. Experiments differ from surveys in that you can control the conditions or variables rather than just collecting data.

Example 1

Design an experiment to see if there is a relationship between students' school marks and the amount of television they watch.



Solution

You need to set up a situation where you can reduce the influence of other factors as much as possible. Factors such as age, I.Q., and grade level should be the same or similar. Once the sample has been chosen, keep track of each student's marks and the amount of time spent watching television every day.

When planning any experiment, you should be very clear as to what your purpose is. Choose the two variables that you are going to measure, and set up the plan that your experiment is going to follow.

2. Ms. Coulson teaches a Grade 9 math class at Midtown Junior High School. She wants to show the students how their work habits influence their grades.

   a. How might Ms. Coulson design an experiment to see if there is any relationship?

   b. What things would Ms. Coulson have to be cautious of or careful with when setting up her experiment?

3. **a.** How could you check to see if there is a relationship between a person's height and his or her shoe size?

   b. What other factors might influence the data?

Check your answers by turning to the Appendix.

It is not an easy task to design an experiment that will explore the relationship between only two variables. There are always other factors that will have some influence on the results. You have to be aware of these influences when you are designing an experiment and when you are analysing the results.

4. Pose a question that will be the basis of an experiment exploring the relationship between two variables for each of the following topics.

   a. jogging **b.** smoking

   c. fertilizing **d.** studying

5. What two variables would you be measuring in order to answer each of the following questions?

   a. Do people who receive flu shots have fewer incidents of flu?

   b. Do people who weigh more at birth live longer?

6. Gather data that could be used to explore each of the following relationships and organize it in a table. Outline how you found the data.

   a. Is there a relationship between the height of a person and his or her hat size?

   b. Is there a relationship between a person's weight and the size of his or her wrist?

   c. Is there a relationship between the circumference and the diameter of a circle?

7. The following newspaper article discusses a relationship between two variables.

Better numbers in girls' class

To sum it up, math scores in a new girls-only class at St. Thomas More Catholic junior high school are improving.

The improvement in the girls' marks has ranged from five to 20 per cent since the exclusive Grade 9 class was set up in October, said principle Brenda Willis.

The findings back research that suggest girls will score better in math and sciences without boys in the classroom.

"The guys aren't there to bug you," said 14-year-old Suzanne Kisteleki, who went from a 65 per cent average in her mixed class at the start of the school year to an 80 per cent score on her latest report card.

"It's a lot quieter," said Kisteleki. "There's no rude comments."

Willis convinced the parents' council at St. Thomas More, 9610 165 St., to separate two Grade 9 math classes into two single-sex groups. The move was made in late October, after first report cards.

One other Grade 9 class was left mixed with boys and girls. The mixed class and the boys-only group have since shown no noticable change in marks.

But things began adding up in the girls-only class and interest is compounding.

The girls-only class is taught by a male teacher.

Willis is now planning separate math classes for girls and boys in Grades 7, 8, and 9 next year.

[1]

   a. Identify the two variables and the relationship that is being discussed.

   b. What conclusion, if any, has been drawn?

8. Design and conduct an experiment to investigate the relationship between two variables. Make a chart to record your bivariate data, and analyse the results of your investigation.

Check your answers by turning to the Appendix.

[1] Timothy le Riche, "Better Numbers in Girls' Class," *The Edmonton Sun*, 5 May 1995. Reprinted by permission.

Now Try This

Use one of the problem-solving strategies to solve the following problem.

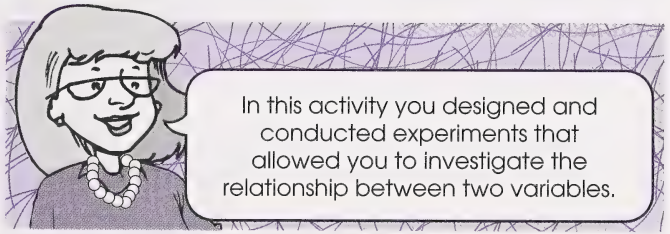
9. A square piece of paper is folded in half (as shown on the right). The perimeter of each new rectangle formed is 36 cm.

a. What is the perimeter of the original square?

b. If the perimeter of each new rectangle was 30 cm, then what would be the perimeter of the original square?

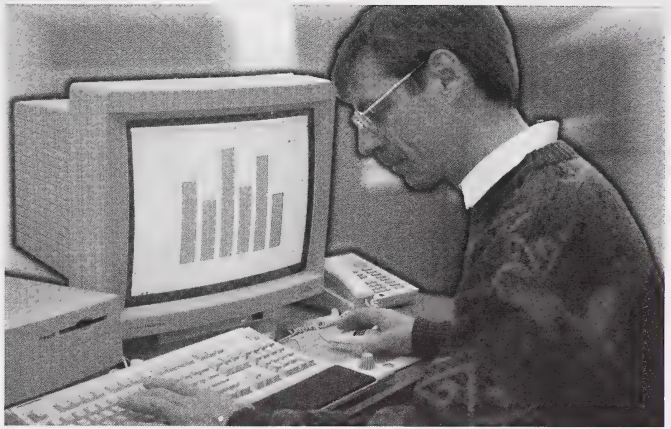
c. Is there a relationship here? Explain.

Check your answers by turning to the Appendix.



Activity 2: Making a Scatter Plot

Anybody who conducts an experiment must devise some way of analysing the data in order to discover any relationships or come to any conclusions.



Statistical data can be analysed through manual calculations or using a calculator or a computer. Many of today's statistics are analysed using computers.

You have become familiar with a number of ways of displaying data from your previous work with statistics. In this activity you will display data involving two variables using a **scatter plot**.

A scatter plot is a graph of a set of points representing the relationship between two sets of numbers or data. The points are plotted like ordered pairs, and the axes of the graph are labelled appropriately.

When the data is plotted on a scatter plot, patterns may become evident that are not clear or easily recognizable when examining the data alone.

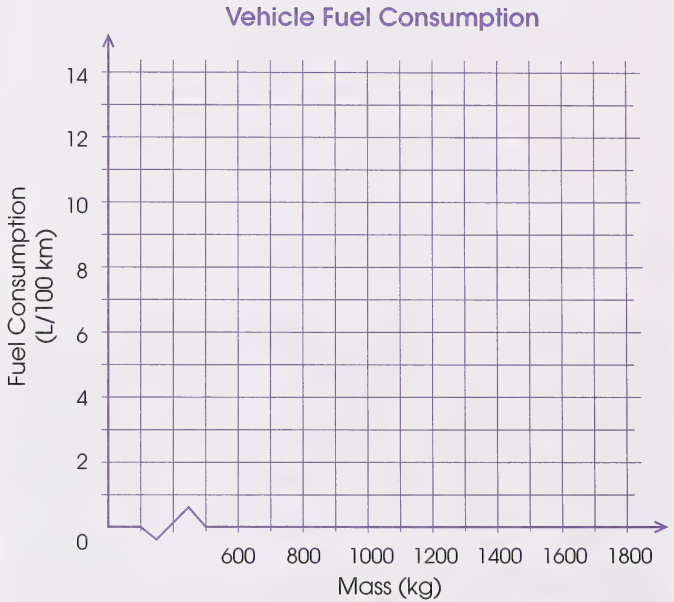
Example

The following data has been collected to determine how the mass of a vehicle affects fuel consumption. Make a scatter plot using the given data.

Vehicle Fuel Consumption			
Mass (kg)	Fuel Consumption (L/100 km)	Mass (kg)	Fuel Consumption (L/100 km)
850	5.2	1100	7.5
1200	8.6	1750	12.3
900	6.3	1400	10.5
1050	8.2	1700	12.0
1300	7.9	1000	8.6
600	4.2	1200	8.2
1500	9.8	900	5.7

Solution

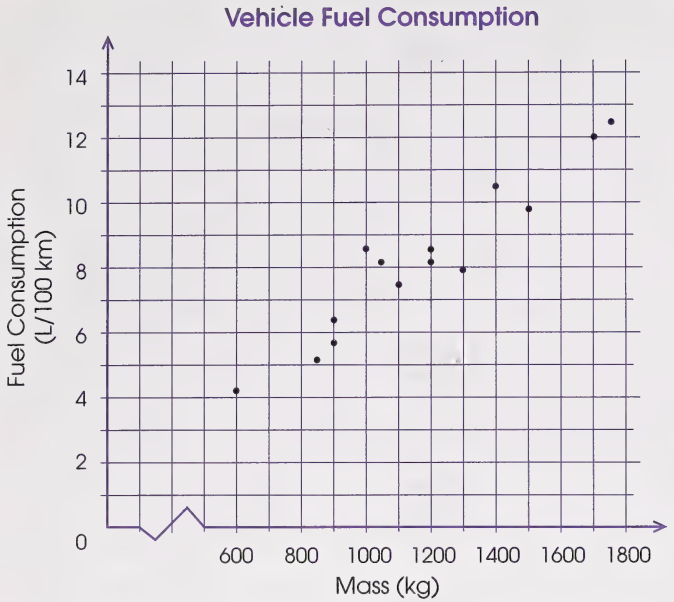
Plot the data on a grid similar to the following.



Fuel consumption is dependent on the mass; so, place fuel consumption (the dependent variable) on the vertical axis and mass (the independent variable) on the horizontal axis.

Always place the dependent variable on the vertical axis and the independent variable on the horizontal axis.

The scatter plot will look as follows.



Note: Always make sure your scatter plot has a title and that the axes are labelled appropriately.

1. Describe the relationship between fuel consumption and mass.

Check your answer by turning to the Appendix.

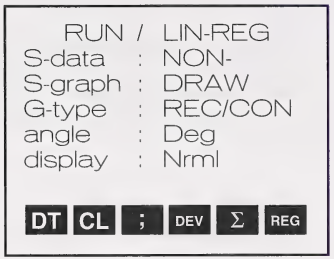
You can also draw a scatter plot using a graphing calculator or a computer spreadsheet program.

Drawing a Scatter Plot Using a Graphing Calculator

If you have access to a graphing calculator, you can create the scatter plot in the preceding example.

The following steps show how to enter the data and create a scatter plot using a Casio® *fx-7700G* graphing calculator. If you have a different brand or model of graphing calculator, consult the graphing calculator's owner's manual to help you modify the steps.

Step 1: Set the modes. The modes should be set like the ones shown in the display to the right. If the modes on the display of your graphing calculator are different, then enter the following keystrokes.
Note: Only enter the keystrokes of the modes you need to change.



- System Mode and Calculation Mode (first line)

MODE 1 MODE 4 MODE ÷

- Stat Data Mode (S-data)

  MODE Shift 2

- Stat Graph Mode (S-graph)

  MODE Shift 3

- Graph Mode (G-type)

  MODE Shift + MODE Shift 5

- Unit of Angular Measurement (angle)

  Shift 1 (DRG) F1 EXE

- Display Mode (display)

  Shift 2 (DISP) F3 EXE

Step 2: Enter the range for the graph. Press Range, and enter the following values. **Note:** Press EXE in order to move to the next line; then press Range once you have entered the values. This will return you to the menu in Step 1.

Xmin:	0	Ymin:	0
max:	1800	max:	14
scl:	200	scl:	2

Step 3: Clear the statistical memory by pressing the following:

Shift 3 (CLR) F2 EXE

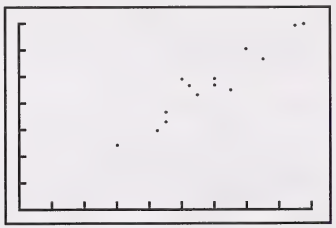
Step 4: Input the following data into the graphing calculator.

$(850, 5.2)$	$(600, 4.2)$	$(1700, 12)$
$(1200, 8.6)$	$(1500, 9.8)$	$(1000, 8.6)$
$(900, 6.3)$	$(1100, 7.5)$	$(1200, 8.2)$
$(1050, 8.2)$	$(1750, 12.3)$	$(900, 5.7)$
$(1300, 7.9)$	$(1400, 10.5)$	

To enter $(850, 5.2)$, enter the following keystrokes. Use this sequence as a guide for entering the remaining data. **Note:** Only press PRE once. Do not press this button before every entry.

PRE 8 5 0 Shift → 5 • 2 F1

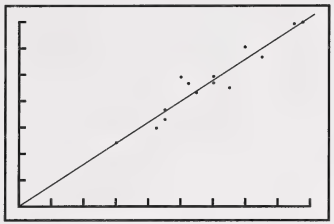
After all the points are entered, the display should look as follows.



Step 5: Graph the line of best fit by entering the following keystrokes.

Graph Shift F4 1 EXE

The display should look as follows.



Note: You will study line of best fit in more detail in Activity 3 of this section.

Drawing a Scatter Plot Using a Computer Spreadsheet

If you have access to a computer spreadsheet program, you can draw a scatter plot using the computer. The following instructions are for *ClarisWorks*™, but you can use any spreadsheet program. You may have to modify the steps somewhat.

Step 1: Access the computer spreadsheet program and enter the data given in the previous example. Enter the data for mass in column A and for fuel consumption in column B.

Your entered data should look as follows.

	A	B
1	850	5.2
2	1200	8.6
3	900	6.3
4	1050	8.2
5	1300	7.9
6	600	4.2
7	1500	9.8
8	1100	7.5
9	1750	12.3
10	1400	10.5
11	1700	12
12	1000	8.6
13	1200	8.2
14	900	5.6

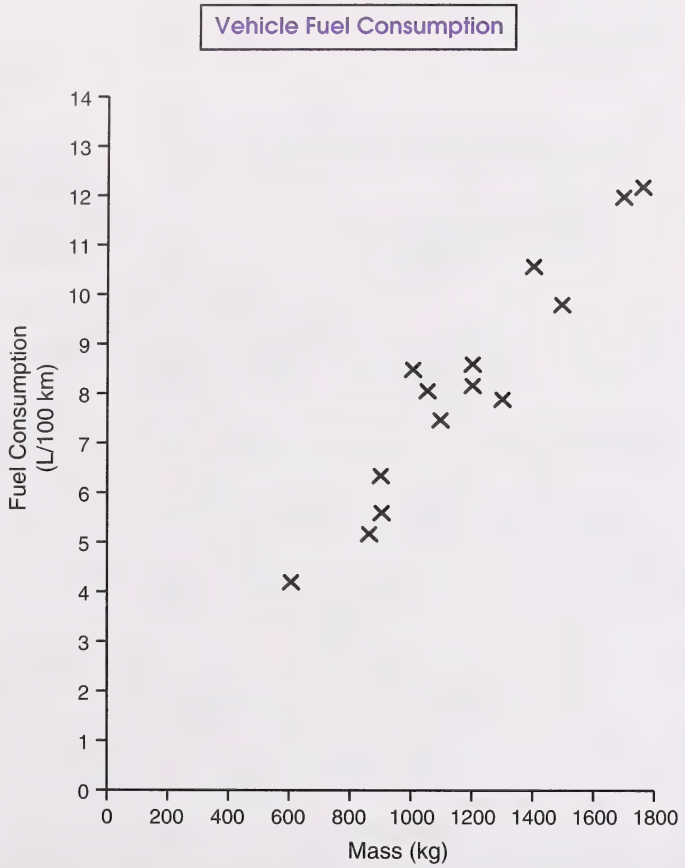
Step 2: Select cells A1 to B14. Click and hold on cell A1, and drag through to cell B14; then release the mouse button. The selected cells should be highlighted except for cell A1.

Step 3: Under the Options menu, choose Make Chart.

Step 4: Double click on the X-Y Scatter icon. You will get a scatter plot immediately.

Step 5: To modify the scatter plot, choose Modify Chart under the Options menu. You can now modify the scale of each axis, add labels to the axes, and give the scatter plot a title.

Step 6: You can change the size of your graph by clicking on the lower right corner and dragging it larger or smaller. Your completed scatter plot might look something like this.



Be sure to save your scatter plot for later use. Refer to the user's guide for additional help if required.

Use either paper and pencil, a graphing calculator, or a computer spreadsheet program to answer the following questions.

2. The coach of the Manning Marauders in the Northern Alberta Senior Hockey League recorded the total number of points scored by each player and the average number of minutes played per game. The results of the first half of the season are listed.

Player Number	Number of Points Scored	Amount of Playing Time Per Game (min)
2	12	27
3	18	15
5	32	24
8	20	18
9	9	10
11	30	20
14	13	12
17	48	28
23	28	20
35	36	26

42	21	16
66	27	19
88	35	22
99	54	32

a. Construct a scatter plot of the data.

b. Describe any relationship or trend shown in the graph.

c. Are there any points that don't seem to fit? If so, give an explanation as to why they don't fit.

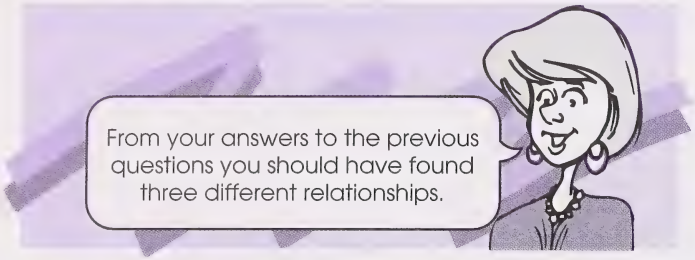
3. Make a scatter plot of the following data, and note any trend.

Daily High Temperature and Relative Humidity			
Temperature (°C)	Relative Humidity (%)	Temperature (°C)	Relative Humidity (%)
20	65	25	50
30	16	10	40
16	85	29	62
18	24	15	32
24	10	9	15
8	78	22	58

4. Make a scatter plot of the following data, and note any trend. Comment on the trend you notice.

Average Monthly Temperature of Two Cities	
Melbourne, Australia (°C)	Washington, D.C., U.S.A. (°C)
26	7
26	7
24	13
20	18
17	24
14	28
13	30
15	29
17	26
19	19
22	13
24	7

Check your answers by turning to the Appendix.



Trends or Relationships in Scatter Plots

- Points plotted rise to the right.
- Points plotted fall to the right.
- Points show no apparent relationship.

5. What type of relationship would you expect for each of the following?

a. age of spouses

b. number of years of post-secondary education and income

c. hat size and sense of humour

d. temperature outside and amount of ice cream sold

e. speed travelled and time taken to reach a destination

Now Try This

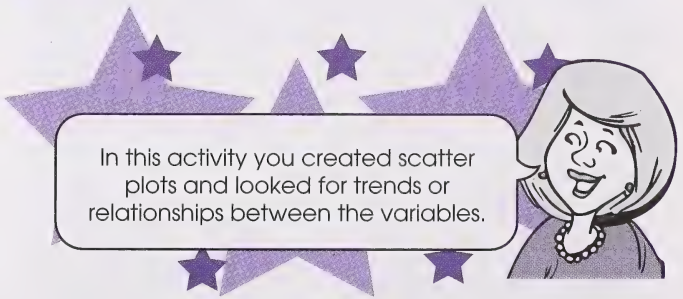
Use one of the problem-solving strategies to solve the following problem.

6. **a.** Find the general rule for the following pattern; then write the next row.

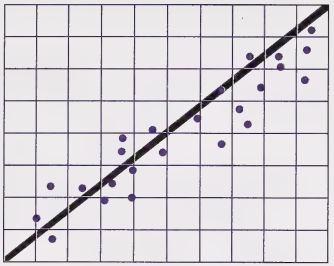
Row 1:	2	2	2						
Row 2:	2	4	6	4	2				
Row 3:	2	6	12	14	12	6	2		
Row 4:	2	8	20	32	38	32	20	8	2

b. What do you notice about each row after the first row?

Check your answers by turning to the Appendix.



Activity 3: Drawing Lines of Best Fit and Analysing Scatter Plots



When a researcher creates a graph from research data, he or she will often connect the points to form a line or draw a line through as many points as possible.

In the last activity you looked at scatter plots to see if there were any general trends or relationships. In this activity you will look at how you can make further interpretations from a scatter plot by adding a straight line to the diagram.

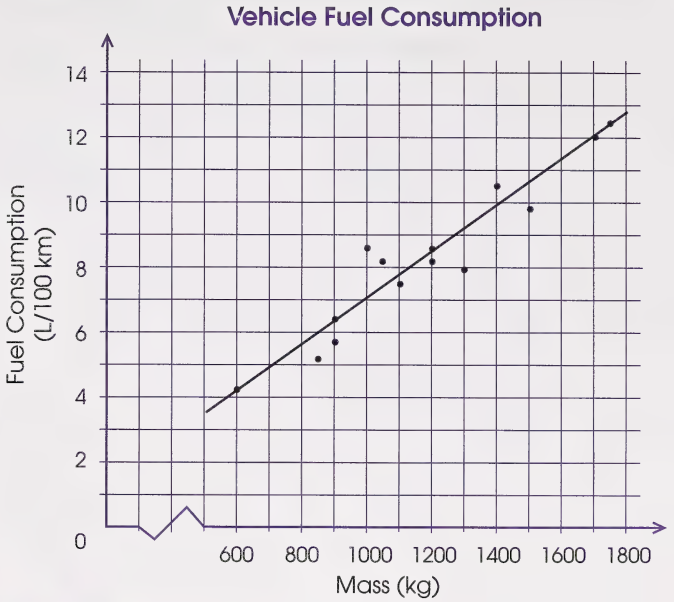
Lines of Best Fit

The points on a scatter plot do not fall exactly along a line, but often a line can be drawn that closely approximates the data. This line is called the **line of best fit**.

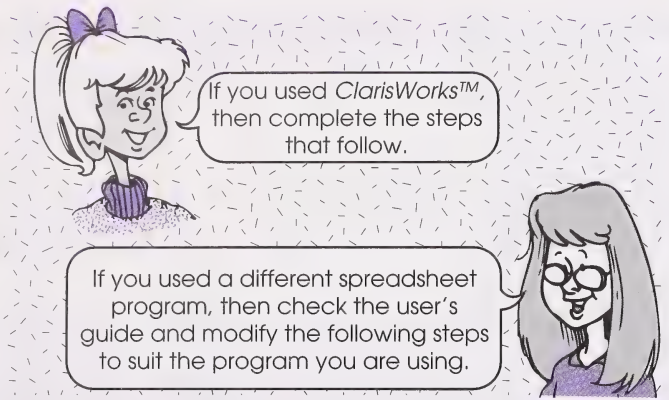
A line of best fit can be drawn by placing your ruler on the graph and drawing a line that appears to best approximate the points on the scatter plot.

Copy the scatter plot you completed for vehicle fuel consumption in Activity 2.

Use a ruler to draw a line through the points so that approximately half are on one side of the line and half are on the other side of the line. You now have a line of best fit for the data. Compare your line of best fit for vehicle fuel consumption to the one shown on the following scatter plot.



If you used a spreadsheet program to create the scatter plot in Activity 2, you can use that same program to add a line of best fit to your scatter plot.



Open the file you saved from Activity 1, and follow these steps.

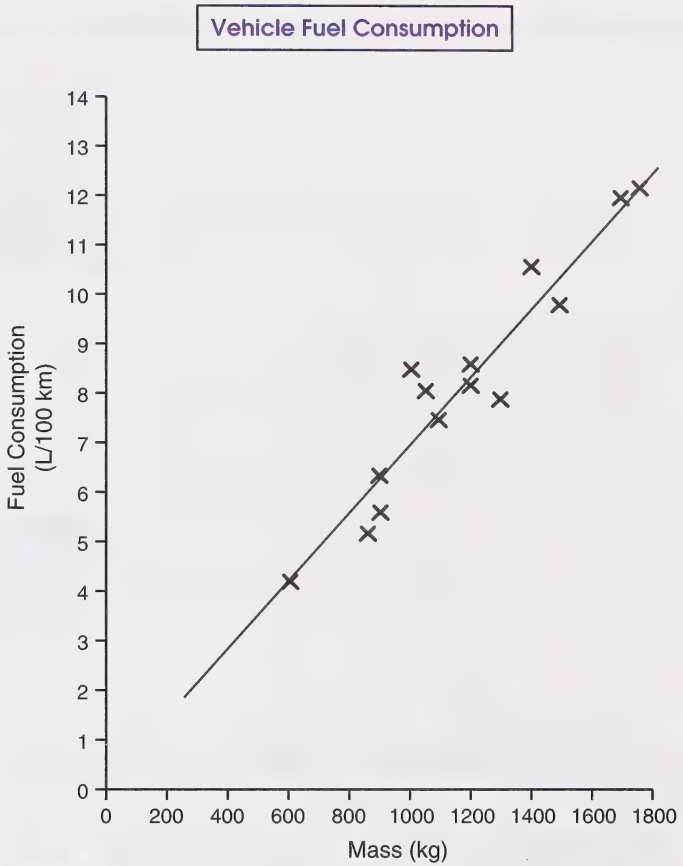
Step 1: Click on View, and select Show Tools. This will show a tool palette on the left side of the screen.

Step 2: Click the small box in the tool palette which contains a line. This will change your pointer to a cross hair.

Step 3: Move the cross hair to one end of your scatter plot data, click and drag through approximately the middle of the points to the other end and release. You should have a straight line through the points.

Step 4: You can adjust the position of the line by clicking on the black dot at either end and dragging.

The following is a sample of a line of best fit for the scatter plot that was created in the example in Activity 2.



If you are using a graphing calculator, refer to the line of best fit you created in Activity 2. You may have to re-enter the data and redo the line of best fit.

1. Is the line a fairly good approximation of the points that were plotted?

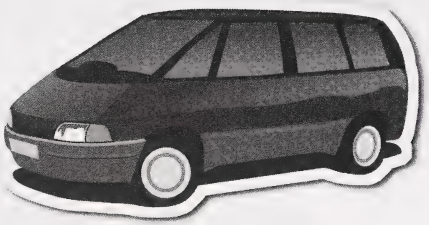
2. What kind of relationship does the line suggest?

Check your answers by turning to the Appendix.

You can use the line of best fit to find values between the given values (**interpolation**) or before the first value and after the last value (**extrapolation**). This is useful in predicting new data based on known data.

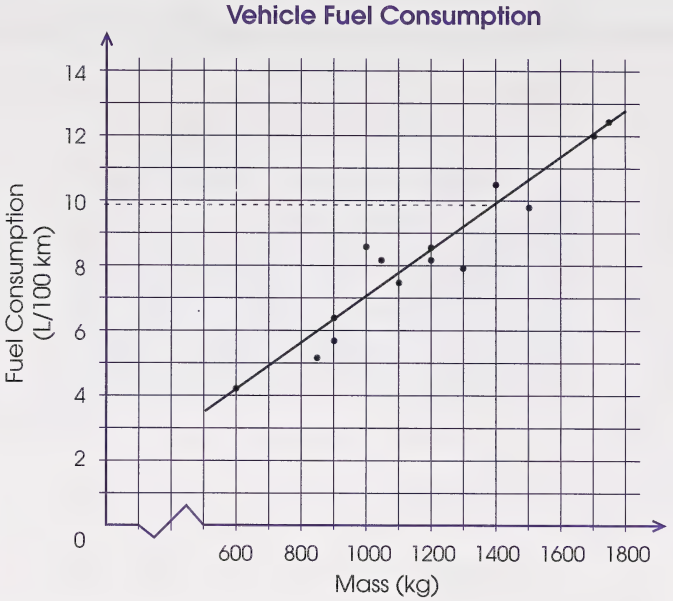
Example 1

Use the preceding graph to find the fuel consumption for a minivan that has a mass of 1400 kg.



Solution

Use a ruler to find the point on the line of best fit that is directly above 1400. Go horizontally from that point to the vertical axis. Read the fuel consumption from the vertical axis.



Therefore, the fuel consumption for a 1400 kg vehicle is about 9.8 L/100 km.

3. Use the graph of vehicle fuel consumption to answer the following questions.

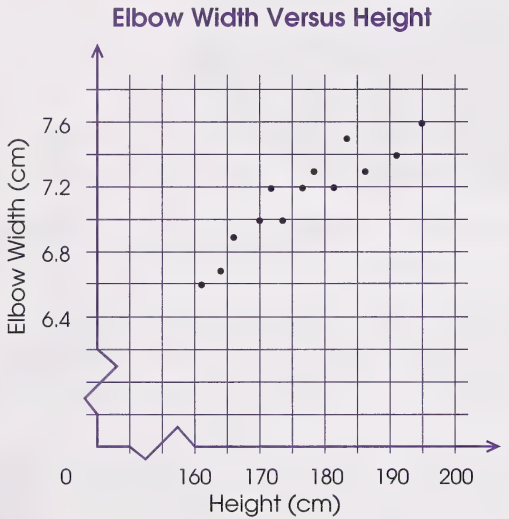
   a. Find the fuel consumption for a vehicle with a mass of 1900 kg.

   b. According to the scatter plot, what would be the mass of a vehicle if it has a fuel consumption of 3.5 L/100 km?

c. Redraw the graph so there is no break in the horizontal axis, and extend the line of best fit to the axis. Does the line cross the axis at the origin $(0, 0)$? Is this reasonable in the context of the two variables used (fuel consumption and mass of vehicle)? Explain.

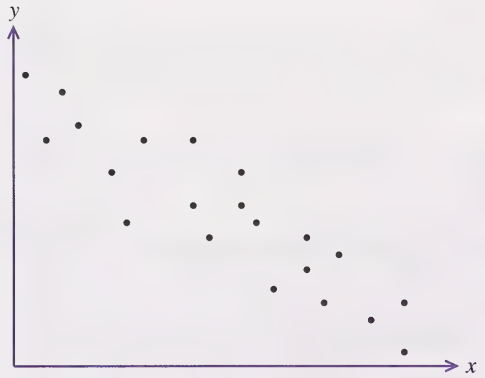
d. What can you say about the fuel efficiency of the vehicles represented by the points that are not on the line of best fit?

4. Copy the following scatter plot of elbow width versus the height of a person, and draw a line of best fit.



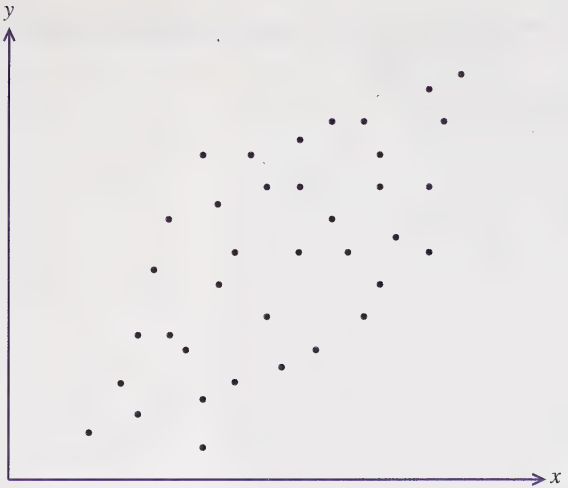
5. Copy the following scatter plots in your notebook; then draw in a line of best fit for each.

a.



b.

c.



6. On which scatter plot in question 5 was the line of best fit easiest to locate? Why?

7. The data in the following table shows the age and number of push-ups that could be done by 20 female teachers at Eastside High School.

Age	Number of Push-ups	Age	Number of Push-ups
24	18	60	1
28	14	45	6
54	4	36	12
48	8	53	2
37	11	40	7
30	15	29	12
34	12	39	21
42	6	43	8
46	5	57	3
51	4	47	5

a. Draw a scatter plot of the data; then draw a line of best fit.

b. What conclusions can you draw from the scatter plot?

c. What might account for the one exception?

d. Did you allow the one point that was an exception to influence your line of best fit?

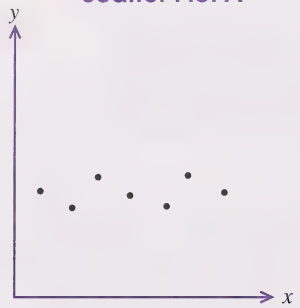
e. According to the graph, how many push-ups can female staff at Eastside High School that are age 60 and over do? Is this necessarily true?

Check your answers by turning to the Appendix.

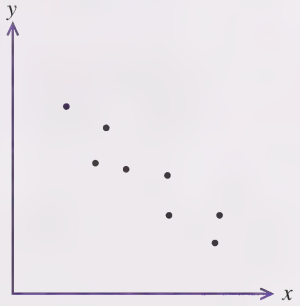
When performing experiments, scientists frequently take measurements to learn how different quantities are related. They plot these quantities on a grid. The resulting graph is a scatter plot.

The following scatter plots are the result of data that was collected.

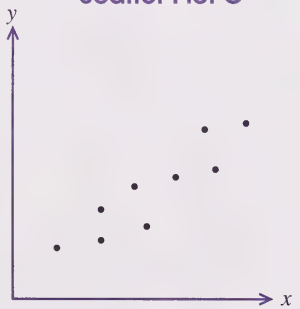
Scatter Plot A



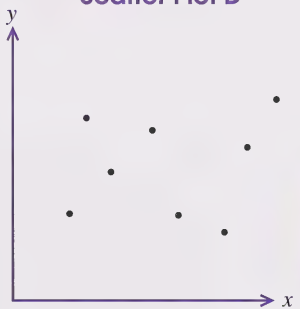
Scatter Plot B



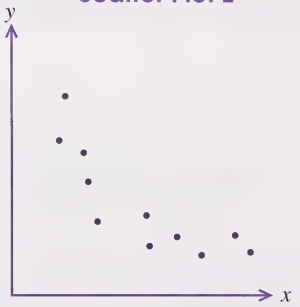
Scatter Plot C



Scatter Plot D



Scatter Plot E



8. a. Which of the preceding scatter plots show some kind of trend?

b. Which scatter plots show a linear relationship between the two variables?

c. What kind of line would you draw through the points in Scatter Plot E?

d. Copy Scatter Plot D, and connect the points. What would you conclude about this data? What would you conclude about the relationship between the variables?

Check your answers by turning to the Appendix.

9. Design an experiment to measure the height a ball bounces versus the height from which it was dropped. Obtain data for at least ten different heights.

a. Make a chart and plot the points on a scatter plot.

b. What trend or relationship do you observe from the line of best fit?

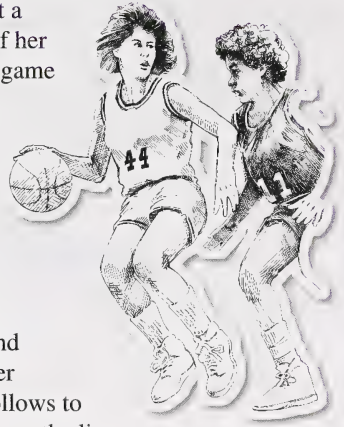
c. Why might some points be noticeably further from the line of best fit than others?

d. Use your line of best fit to predict how high a ball dropped from 100 mm would bounce.

e. Will the type of ball (such as a tennis ball, a rubber ball, or a superball) make a difference?

f. If possible, test your prediction in question 9.e. using different balls.

10. Karen, a basketball coach, kept a number of statistics about all of her players for each game in a ten-game period. After the tenth game, Karen analysed the data carefully to see what she might do to improve her team's performance.



a. Two variables that Karen tabulated were the amount of time played per game and number of points scored per game. Use the data that follows to make a scatter plot; then draw the line of best fit. Label each axis appropriately, and give the scatter plot a title.

Player	Time Played (min)	Number of Points Scored
Muldoon	4	3
Zapisocki	7	4
Koupax	10	6
Reusel	12	10
Deen	12	6
Bouvier	16	11
Smithson	17	8

Armdal	17	14
Leader	20	11
House	20	20
Wong	22	14
Rav	24	18
Naidoo	27	18
Ryning	29	12
Wetland	29	22

b. What is the general trend for the scatter plot?

c. What questions might Karen, as a coach, be asking herself about players Ryning and House based on the data in the scatter plot?

d. If a player has an average playing time per game of 25 min, how many points would you expect her to score?

e. If one of the players' playing time was increased to 30 min per game, how many points might you expect her to score?

Check your answers by turning to the Appendix.

Now Try This

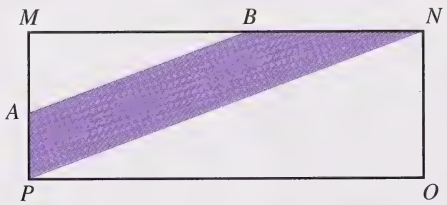
Use one of the problem-solving strategies to solve the following problems.

11. Four strips of cardboard, 30 cm by 3 cm, are arranged to form a square.

a. What is the area of the inner square?

b. What is the area of the outer square?

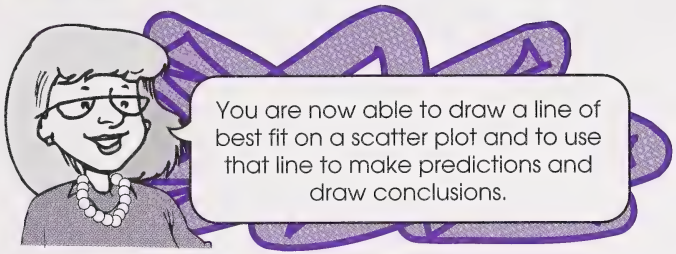
12. A and B are the midpoints of the two sides of rectangle $MNOP$. What fraction of the rectangle is shaded?



Word Exploration

13. If you use the letters from the word "relationships," you can form many other words. Your challenge is to see if you find at least three words of five letters or more beginning with each of the letters in the word *relationships*. You must use only the letters that are in the word. For example, the word *splint* can be formed.

Check your answers by turning to the Appendix.



Follow-up Activities

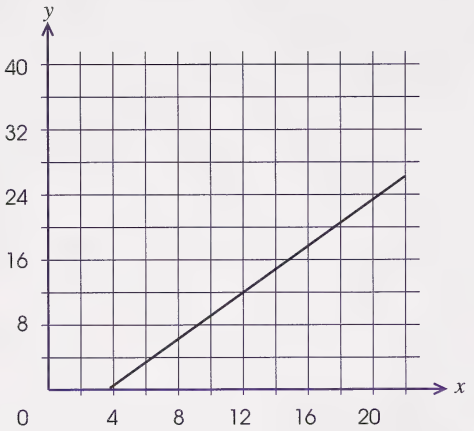
If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

In this section you interpolated and extrapolated information from a scatter plot. To read information from a line of best fit or any line on a graph, you have to match the reading on the horizontal axis with the vertical axis.

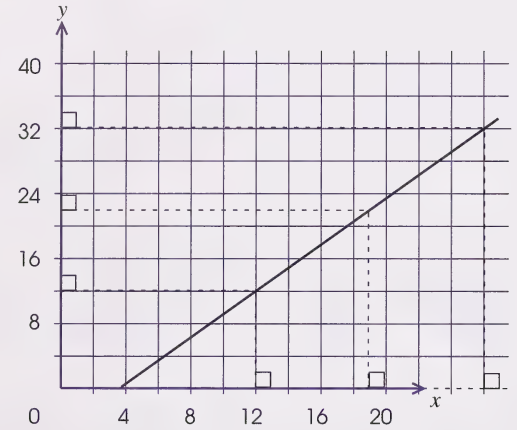
Example 1

Use the graph given to find the y-value that corresponds to each of the following x-values: $x = 12$, $x = 19$, and $x = 26$.



Solution

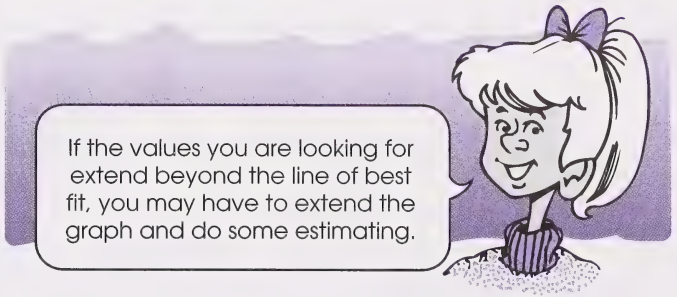
Draw a vertical line straight up from each of the x-values until it meets the line of best fit. Then draw a horizontal line from the point where the vertical line meets the graph across to meet the y-axis. This is the y-value that corresponds to the given x-value. **Note:** You will have to extend the x-axis and the line on the graph to find the corresponding y-value for $x = 26$.



If $x = 12$, then $y = 12$.

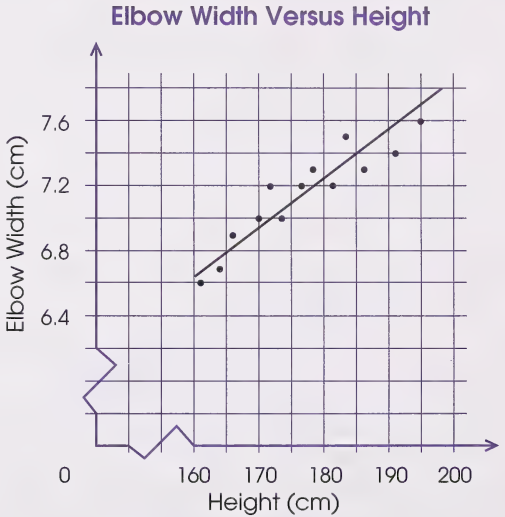
If $x = 19$, then $y = 22$.

If $x = 26$, then $y = 32$.



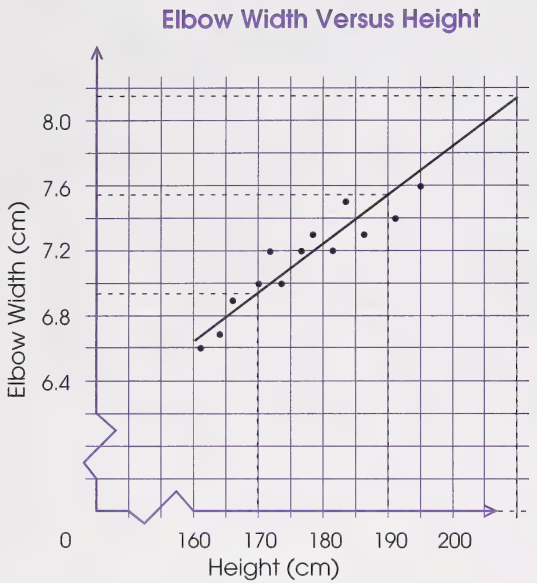
Example 2

Use the line of best fit on the following scatter plot to find the expected elbow width for each of the following heights: 170 cm, 190 cm, and 210 cm.



Solution

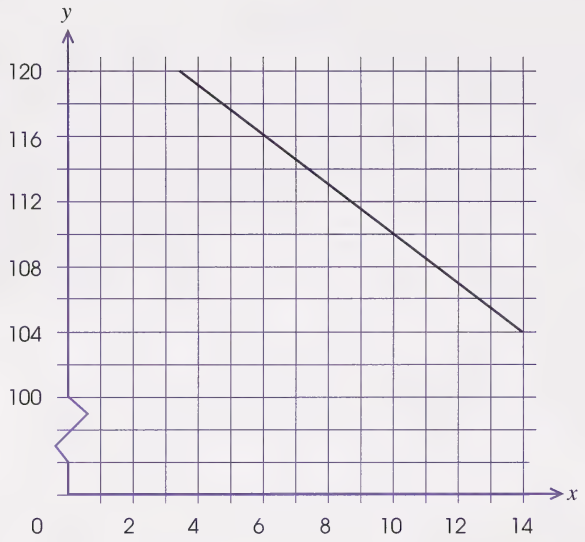
Draw the vertical lines up from the indicated heights (extending the axis where necessary), and find the matching points on the Elbow Width axis. Read the elbow width to the nearest tenth of a centimetre.



The following chart gives the matching points.

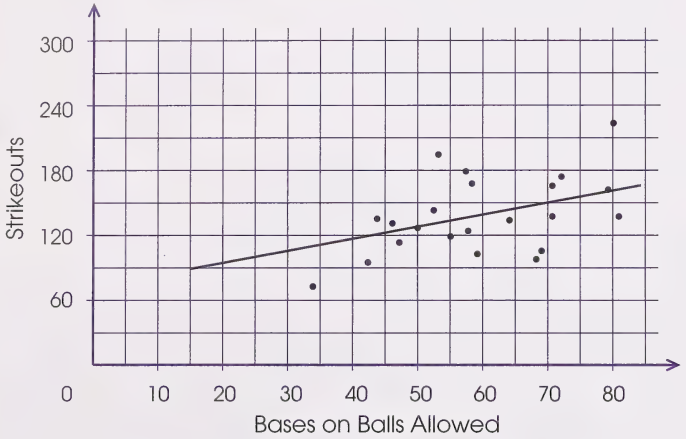
Height (cm)	Expected Elbow Width (cm)
170	6.9
190	7.6
210	8.2

1. Use the graph given to find the y-values that correspond to the following x-values.



a. $x = 6$ **b.** $x = 12$ **c.** $x = 16$

2. The following scatter plot and line of best fit shows the number of strikeouts for the number of bases on balls allowed for a particular pitcher.



Predict the number of strikeouts for the following number of bases on balls.

a. 30

b. 70

c. 90

Check your answers by turning to the Appendix.

Enrichment

1. Design and conduct an investigation on one of the following relationships. Use a scatter plot and a line of best fit to analyse the results of your investigation.

- spring extension versus mass
- mass versus volume for a given substance
- Canadian book prices versus American book prices

a. Describe the pattern of the dots on your scatter plot.

b. Account for the dots that do not lie on the line of best fit.

c. State a relationship for your data.

d. What kind of predictions can you make from the line of best fit?

2. Design and conduct an investigation on temperature versus time of day over a two-day period. **Hint:** Use 2-h time periods from zero to 48 since there are 48 h in two days.

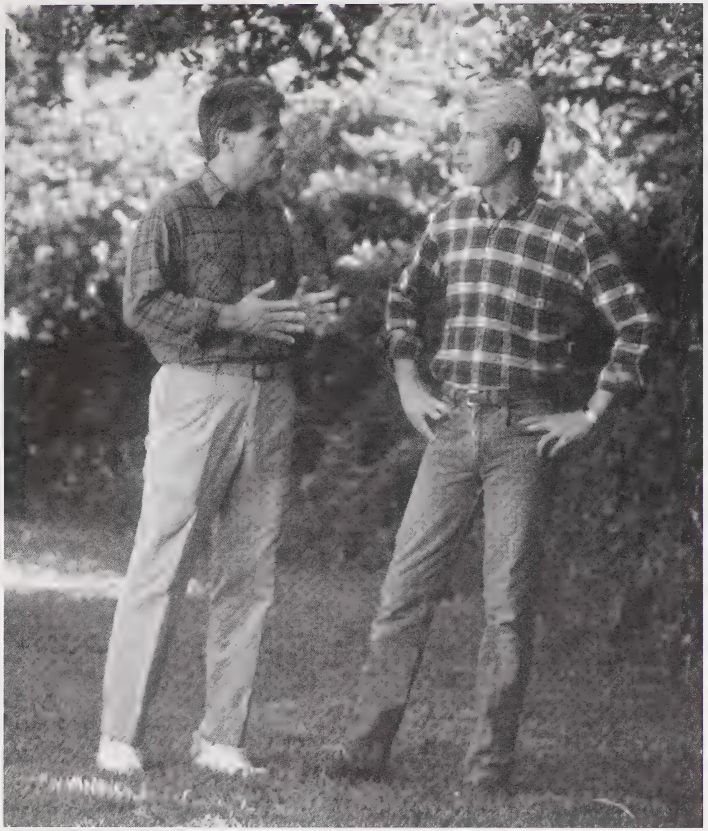
a. Plot the data.

b. Draw a line of best fit through the points.

c. How does the line of best fit differ from the one you drew in question 1?

Check your answers by turning to the Appendix.

Conclusion



In this section you have gone through the first four steps of statistics. You collected data from an experiment you designed and organized, then presented that data using scatter plots. You also looked at ways to analyse data by drawing a line of best fit. Finally, you used scatter plots and lines of best fit to draw inferences or conclusions about the variables involved.

By designing an experiment and obtaining a follow-up analysis, you can discover the relationship, if any, between any two variables. You can determine such relationships as the height of a person to the heights of that person's parents.

Do you think you will be approximately the same height as your parents?

Assignment

Assignment Booklet

You are now ready to complete the assignment for Section 1.

Section 2: Use and Misuse of Data

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Have you ever answered the phone and been asked to answer some questions about a particular topic? If so, did you participate or decline? Chances are you might have been offered an incentive to take part in the survey.

Telephone surveys are just one method used to collect data or information. Who collects this information and for what purpose?

In this section you will assess the strengths, weaknesses, and biases of different methods of collecting data. You will also investigate how statistics are used and misused by various individuals and groups. You will examine some of the devices used to influence your decisions or to persuade you to follow certain directions.

Activity 1: Fairness in Data Collection

Statistics are used every day by individuals and groups in business, advertising, government, and sports to inform, persuade, and help people make decisions.

July 28, 1997

Three out of every four people surveyed use glow toothpaste

July 29, 1997

Interest rates have dropped for the third consecutive week

July 30, 1997

37 percent of voters support the liberals

You may already be familiar with a variety of methods for collecting data (such as surveys, census, experimentation, and questionnaires). In this activity you will assess the strengths, weaknesses, and biases of various data collection methods by analysing articles and advertisements in newspapers and magazines and on radio and television.

Read the article from *The Edmonton Journal* and answer the questions that follow.

Canadians more stressed out—survey

The Canadian Press

Toronto

When it comes to stress, Winnipeg city worker Sharron Gould knows what she's talking about.

Taking a break from answering phone calls from floodwater-weary residents, Gould briefly assumed her role as president of the Canadian Mental Health Association to discuss a new survey that indicates stress levels are rising across Canada.

About 40 per cent of 890 Canadians interviewed by the association between Feb. 17 and March 24 said they're more stressed now than two years ago.

The poll, released in conjunction with national mental health awareness week beginning Monday, suggested Canadians are worrying most about finances (55 per cent), work (39 per cent), health problems (28 per cent), parenting (25 per cent) and housework (23 per cent).

Flooding is nowhere on the list—which is both heartening and worrisome to Gould, a city human resources employee who counsels workers.

In a phone interview, she surmised problems that can be largely controlled—like balancing a budget, or staying in a job they hate—are flipping out Canadians more than uncontrollable stresses, like the flooding that's driven more than 25,000 southern Manitobans out of their homes.

An inability to control stress can lead to physical health problems, making life pretty miserable.

"Stuff gets in the way of your mental health all the time," said Gould.

"But if you keep good family support and friends, you can cope with things better, like sudden death, the loss of a job and freaks of nature like this flood."

About the flood, Gould responds: "We make plans and God laughs down on us."

So why are so many people so out of control of their happiness?

"Life is generally just more complex," Gould says. "Now there are blended families, both parents working, children with higher expectations to produce. We're looking at complex family structures that we would never have envisioned."

These stressful complexities are more commonly working their way into workplaces, which is all the more reason for employers to help workers any way they can, says Gould.

"It's an illusion that people leave their personal problems at home. To be productive at work, people have to gain control of their personal lives, and the workplace is starting to support that," by offering more counselling, flex hours, and alternative work arrangements.

STRESS SURVEY

- 82 per cent say maintaining good mental health is very important.
- 40 per cent said they felt seriously stressed at least a couple of times a week or more often.
- Among the most stressed are those in the 18-24 and 25-34 age groups (49 and 45 per cent, respectively), and they also lead other age groups in their lack of knowledge about how to gain mental health.
- The most common stressors are financial problems (55 per cent), work (39), health problems (28), parenting and children (25), housework (23), unemployment (22), aging (12), elder care (7), relationships (4) and education (3).
- 42 per cent say the amount of stress in their work and workplace has a negative impact on how well they perform their jobs, while 30 per cent say stress is positive.
- Top ways to relieve stress include talking to others (21 per cent), exercise (18), walking (14), keeping a positive attitude (13), reading (12), physical activity (11), relaxing and meditating (10), music, socializing and sleep-rest (8).

[1]

[1] "Canadians More Stressed Out—Survey," *The Edmonton Journal*, 2 May 1997, A13. Reprinted by permission of The Canadian Press.

1. How was the data collected?
2. Why was the data collected this way?
3. Would you say the data collection method was appropriate for the data and for the issue?
4. Is the information presented clearly and honestly?
5. Who makes up the sample in the article? How can you tell who makes up the sample?
6. Do you think the data collected and resulting conclusions are representative of all Canadians in the indicated group?

Check your answers by turning to the Appendix.

If you look at any newspaper or magazine you will find several articles containing some kind of data. In the article you just analysed, you had to interpret how the data was collected and how this may have affected the data and the conclusions drawn.

Before going into more detail on assessing the strengths, weaknesses, and biases of data collection, answer the following questions on data collection to review what you studied in previous mathematics courses.

7. If you have access to a library or the Internet, research the census of Canadians that is held every few years.

   a. How is the census conducted?

   b. How much money does a census cost?

   c. Why is the census necessary? What is done with the data that is obtained?

   d. Is the census 100% accurate? Why or why not?

8. State some advantages and disadvantages of using each of the following methods for gathering data in a survey.

   a. a face-to-face interview

   b. a telephone interview

   c. a mail-in questionnaire

9. Data collection can be affected by a number of factors. What effect might each of the following factors have on the responses of individuals?

   a. the time of day a phone interview is conducted

   b. Television crews are filming the personal interviews.

   c. the number of questions on the questionnaire

   d. the location of the personal interview

   e. A well-known TV celebrity is conducting the interview.

   f. Respondents to a questionnaire are eligible for a prize.

   g. The phone interview is a computer-generated, push-button response type.

10. Which of the following situations is a random sampling of students from a junior high school? Explain any that are not.

    a. Select all the students whose telephone number ends in a 5.

    b. Assign each student a number and then select numbers from a container that has all the numbers in it.

    c. Select those students whose surname begins with A, G, L, or T.

    d. Assign each student a number; select a card from a pile of cards that have the digits from 0 to 9 on them; then take each student whose number ends in the digit drawn from the pile.

    e. Select 15 students from each grade by listing them alphabetically and selecting every tenth one, twelfth one, and so on as required.

    f. Assign each student from each grade a number and then select one student from each grade using a random digits table.

Check your answers by turning to the Appendix.

Now you will analyse some newspaper articles and advertisements for strengths, weakness, or bias of how data is collected and how the conclusions based on the data are drawn.

Read the article and answer the questions that follow.

City jobless rate defies rising trend
Stats fuel campaign fires

HELEN PLISCHKE
Journal Staff Writer
and Journal News Services
Edmonton

There was good news for Edmonton's jobless Friday with fresh statistics showing the city's unemployment rate dropped nearly a whole percentage point in April.

"This is very promising. I didn't expect to see these gains in only 30 days," said Edmonton Chamber of Commerce general manager Martin Salloum.

Unemployment in Edmonton dropped from 7.9 per cent in March to 7.2 per cent in April. That brings the city closer to Calgary's 6.4 per cent figure, which went unchanged between March and April.

Provincewide, the rate increased slightly to 6.3 per cent, representing 95,900 unemployed.

The number of full-time jobs in Alberta rose by 11,900 in the 12 months prior to April 1997. Part-time jobs increased by 7,500 in the same period.

National statistics weren't so bright, throwing a contentious new issue into the federal election campaign.

An estimated 33,000 more people found jobs last month but a flood of 79,000 workers into the labour market drove the jobless rate up to 9.6 per cent in April, from 9.3 in March, and the number of unemployed back up towards 1.5 million.

The figures seem to contradict the claim Prime Minister Jean Chretien has been making at every stop on his tour—that his government has turned the economy around and created new hope.

The prime minister attributed last month's unemployment rate—9.6 per cent—to growing public optimism that encouraged more people to seek work.

"People have more confidence and people who were discouraged are coming back into the labour force."

Unemployment pushed itself onto the prime minister's agenda in Newfoundland when about 400 jobless protesters—including Buzz Hargrove, president of the Canadian Autoworkers' Union—attempted to derail a campaign event.

Friday's jobs report from Statistics Canada, the last before the election, handed political leaders of all stripes ammunition with which to attack or defend the government's employment-creation track record, which they did.

Provincial jobless rates
April 1997

Newfoundland	19.1%
P.E.I.	16.0%
Nova Scotia	13.3%
New Brunswick	13.6%
Quebec	11.6%
Ontario	9.0%
Manitoba	6.6%
Saskatchewan	6.2%
Alberta	6.3%
British Columbia	9.1%

Seasonally adjusted
Source: Statistics Canada

[1]

11. Is the data presented clearly and honestly? Pay particular attention to the first sentence in paragraph one and the data presented in paragraph three.

12. How are your conclusions regarding Edmonton's jobless rate affected by the statement in the first paragraph?

13. Explain the headline for the article in terms of the statistics.

[1] Helen Plischke, "City Jobless Rate Defies Rising Trend," *The Edmonton Journal*, 10 May 1997, A1. Reprinted by permission.

14. Collect data from a newspaper or magazine article, or from the radio or television. Analyse the data and answer the following questions.

a. How was the data obtained?

b. Were the data collection methods appropriate for the issue involved?

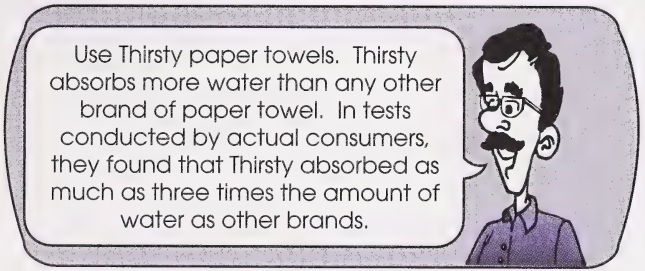
c. How might you collect the data differently?

d. Do the conclusions follow logically from the data?

Check your answers by turning to the Appendix.

Not all data is collected and used clearly or honestly. Sometimes there is a deliberate attempt to use data in a particular way or to leave certain questions unanswered.

The following advertisement appeared on a local television station.



15. How many people do you think might have been used as consumers in the tests?

16. Is there any indication that some other brands may have been close in the amount of water absorbed?

17. Do you see any bias in how the data may have been collected for this advertisement?

18. How could you collect and present the data differently?

Check your answers by turning to the Appendix.

A magazine advertisement states the following:

19. Do you think all dentists in the province were consulted?

20. Is this sample representative of all dentists?

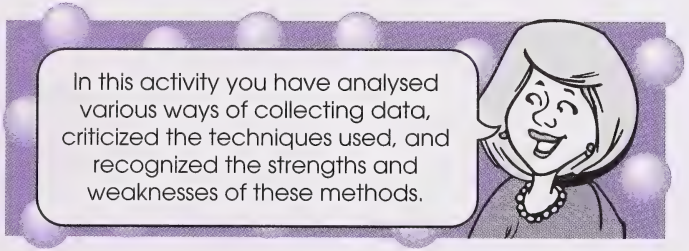
21. Find an advertisement in a newspaper or magazine, or on television. Analyse the advertisement and answer the following questions.

a. Is the data presented clearly? Is there bias in how the data is presented?

b. Do the conclusions follow logically from the data?

c. Are there questions that are left unanswered? Is this deliberate?

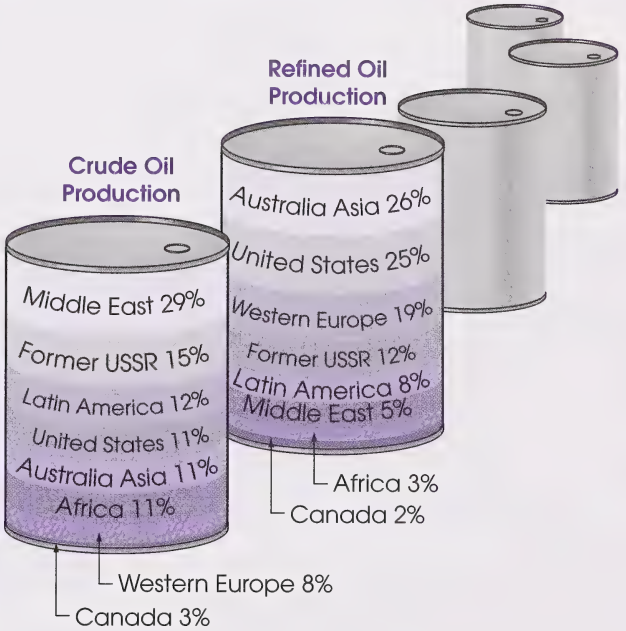
Check your answers by turning to the Appendix.



Activity 2: Use of Statistical Information

Have you looked at a newspaper or magazine recently? How many different ways can you find statistical information displayed?

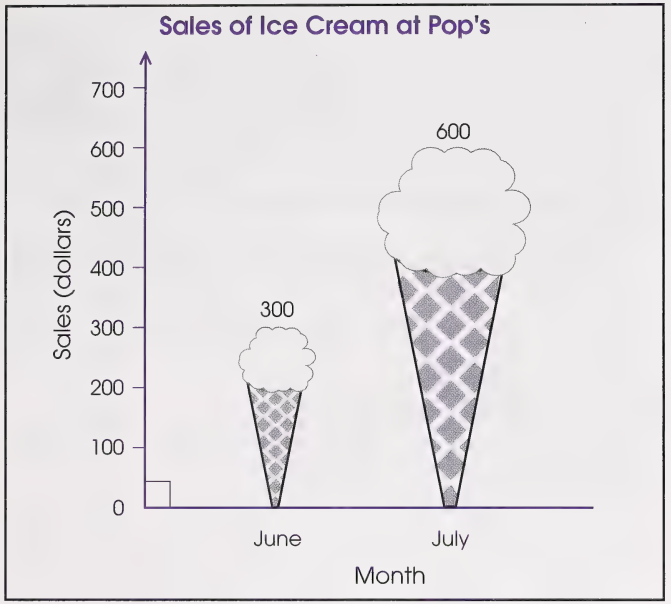
Unique methods are used to help the reader understand the information clearly, or to influence the reader. Sometimes the information helps the reader understand the presentation more quickly, as in the graph involving the oil barrels.



At other times certain information is exaggerated so that the reader gets a particular impression of the information that is presented.

In this activity you will analyse and critique ways in which statistical information is presented by various sources of media.

1. Tanya operates an ice cream franchise called Pop's. She sells $300 of ice cream in June and $600 in July. She makes a graph of June and July ice cream sales to show to the franchise owner.



a. Does the graph present the information accurately? Explain.

b. What impression do you get when you look at this graph?

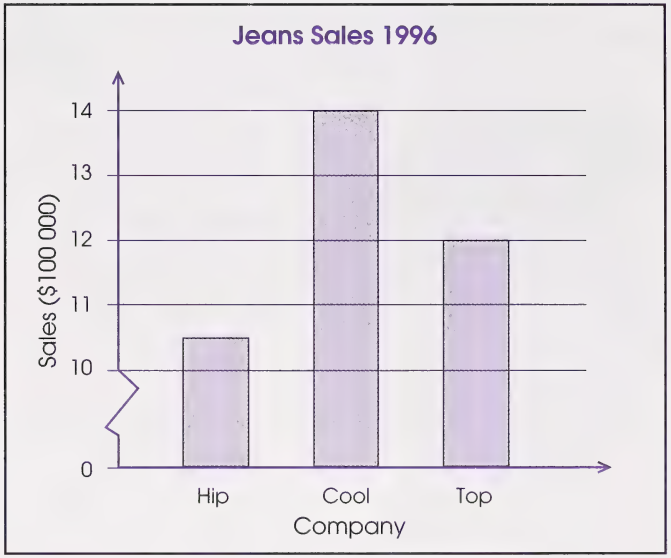
c. How could the graph be improved?

d. What is Tanya's purpose in using the ice cream cones in the graph? Does she achieve her purpose?

e. If you were making this graph, would you change it to one using uniform bars?

2. Companies, sales people, and product promoters often try to present information in ways that portray the product or service in the most favourable way possible without actually presenting false information.

Study the following graph and answer the questions.

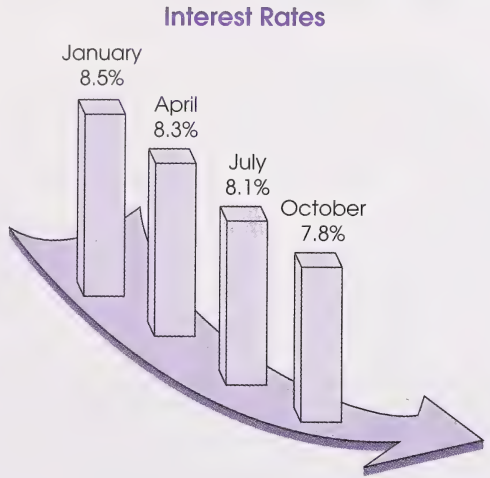


a. How is this graph misleading?

b. What impression would readers likely get when first looking at the graph?

c. Which company would be likely to present the data in this manner?

3. A newspaper article has the following graph included.



a. Why is this graph misleading?

b. Who might be presenting the data in this manner?

c. How could you change the graph to make it reflect the facts more accurately?

Check your answers by turning to the Appendix.

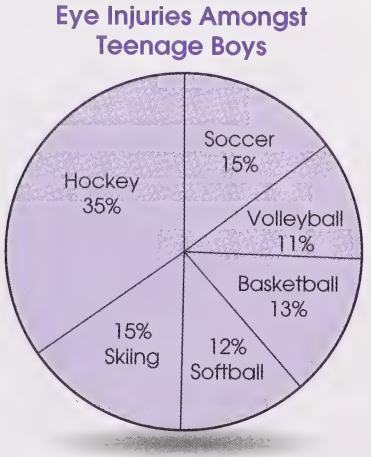
Sometimes you can be influenced by missing information.

4. Sales for a computer store are up 25% over the sales from last year. A television news reporter states that the store is doing very well.

a. Is this necessarily true based upon this statistic? Explain.

b. Who might be affected by the reporter's statement?

5. The following chart is presented in a newspaper article with the heading Hockey the Most Dangerous Sport.



a. Is it fair to conclude from the graph that hockey is the most dangerous sport?

b. Is it fair to conclude that hockey participants need to do something more to protect their eyes?

c. What other information do you need before you can make any real conclusions on the relative dangers of all the sports?

Check your answers by turning to the Appendix.

You cannot believe everything that you read, see on television, or hear on the radio. You have seen examples of information that have not been presented fairly and accurately. In the previous activity you saw how data can be biased because of the sample used, the manner in which the survey is conducted, and the way the question is asked.

You must always look carefully at any claims made by polls, researchers, and so on. How did they get their data? Who did they survey? What information was given? What questions did they ask? What was their motive for conducting the survey?

Example 1

Read the following newspaper headline and article, and comment on the accuracy of the information portrayed.

July 28, 1997

THE REPORTER

Four out of five teenagers have poor teeth

Dr P. White, a local dentist, told a reporter in an interview yesterday that of the 40 teenaged patients he saw in a recent week, 32 of them had to be treated for cavities.

Solution

The headline is inaccurate on several counts. First, it is based only on the patients who went to see Dr. White. They are not representative of the population. Second, the patients most likely went to see the dentist because they were having trouble with their teeth. Third, it does not say anything about the extent of the cavities. Some of them could have been very minor, and therefore the teenagers may not really have poor teeth.

6. Study the following claims and explain how each is misleading. What does it seem to be saying? What might it really be saying?

a. Our French fries have 30% less fat.

b. More people switched to our new line of soft drinks than any other.

7. Recently, you have had the following test results in your mathematics class: May 15 – 59%, May 29 – 67%, and June 5 – 69%.

a. Draw a graph that you might use to show your improvement to your parents.

b. Draw a graph that your teacher might use.

c. Comment on both of them.

8. Give three reasons why an individual, a company, a group, or anyone else might wish to mislead readers, viewers, and others with data.

9. Find a misleading advertisement in a newspaper or magazine, or on the radio or television. Discuss how it might be misleading and how you think it should be changed.

Check your answers by turning to the Appendix.

Now Try This

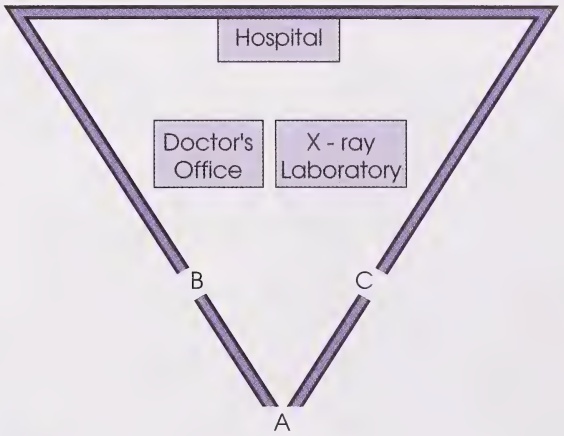
Use one of the problem-solving strategies to solve the following problem.

10. The karat is used to measure the purity of gold and gold objects. Pure gold is rated at 24 karats. What is the karat rating of a gold ring that is 75% gold?

Check your answer by turning to the Appendix.

Pathologically Correct

11. Patient A must get to the hospital, patient B must get to the x-ray laboratory, and patient C must get to the doctor's office. Copy the diagram and draw paths that would get the patients to their respective destinations so that none of the paths cross and all paths stay within the triangle.



Check your answer by turning to the Appendix.

In this activity you learned to recognize misleading graphs and charts and how to view and interpret statistics with a very critical eye.

Follow-up Activities

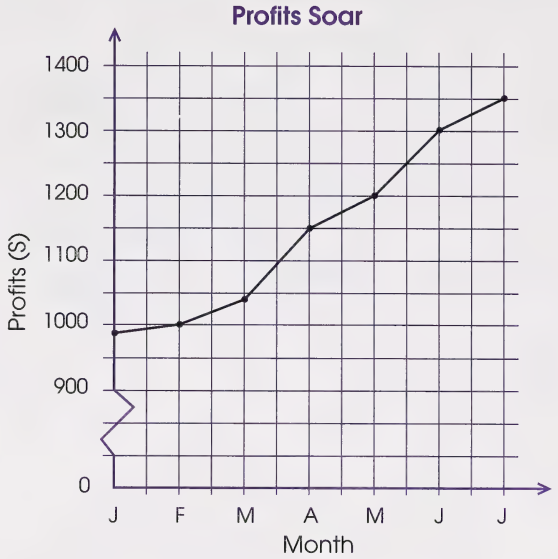
If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

Advertisements try to present data about the sponsor's products in a favourable way and sometimes stretch the truth by creating a false impression of the data. Don't believe everything that you read or see—or what you *think* you read or see.

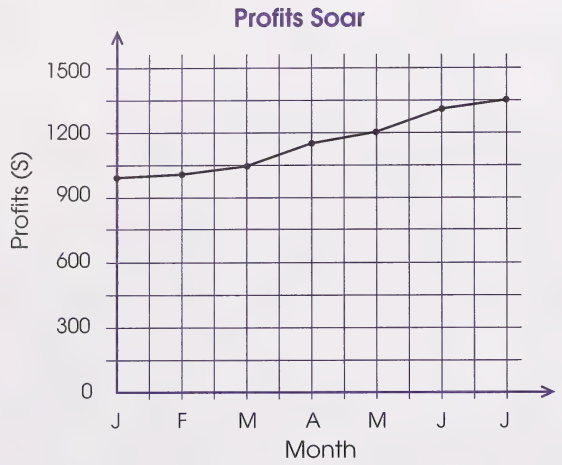
In Activity 2 you studied some graphs that were misleading. Here are some different graphs that are misleading. Study the examples and answer the questions that follow.

Example 1

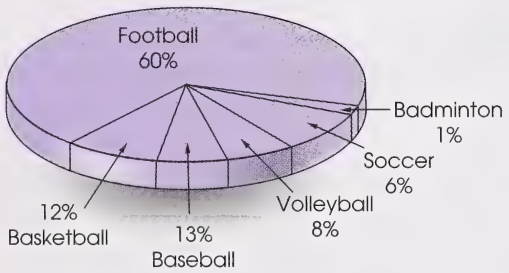


The vertical scale is broken and spread out to give the impression that the profits are rising quickly.

If the scale was not broken and was more compressed, the graph would look like this.

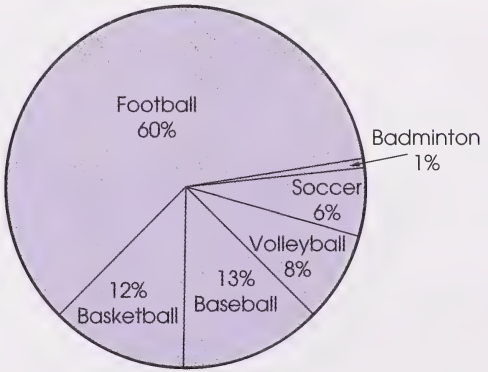


Example 2

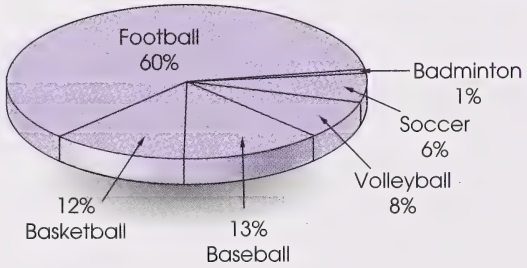


The three-dimensional appearance of the graph is deceiving. It looks as if more of the budget is spent on soccer than on basketball or baseball. The angles are correct but the perspective makes it look like soccer has a larger portion. This is not the case.

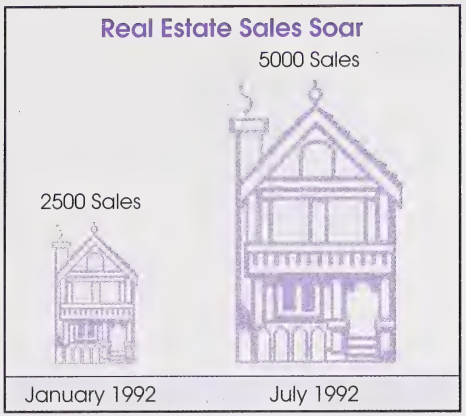
The following two-dimensional graph is more accurate.



The graph could be drawn this way in three dimensions.



Example 3

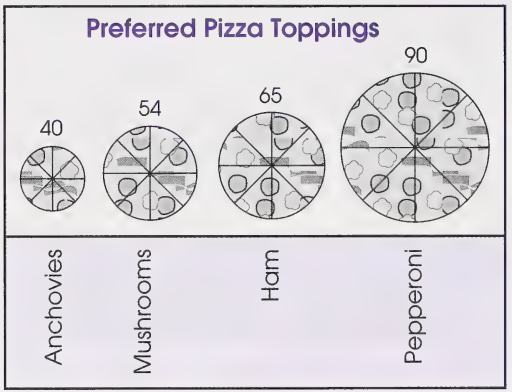


This pictograph is misleading because the areas of the pictures are out of proportion to the sales. The sales in July are twice those in January. However, the picture representing the July sales is twice as tall and twice as wide as the picture representing January sales.

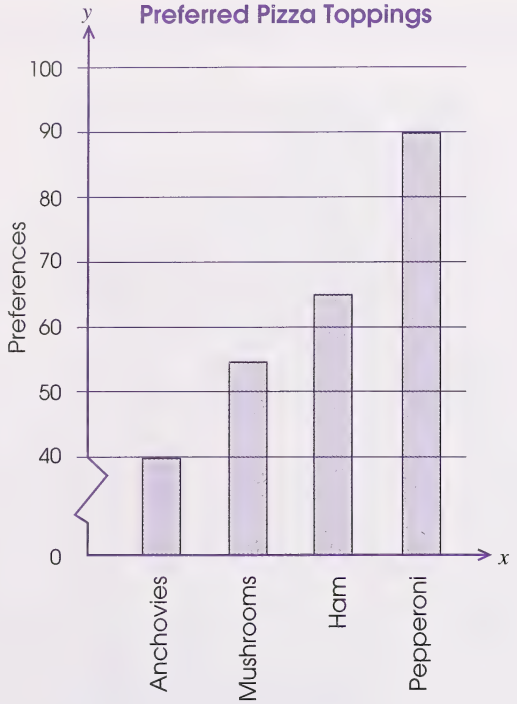
A more realistic graph uses individual pictures which are the same size.

1. Explain why the following graphs are misleading.

a.



b.



2. Redraw the graphs in Question 1 more realistically.

Check your answers by turning to the Appendix.

Enrichment

There are many creative ways to display data. The ice cream cone graph in Activity 2 is one example.

Search through newspapers and magazines to find at least three examples of creative graphs displaying data. Cut out the graphs and paste them on a page in your workbook.

1. Write out how each graph is unique and creative.

2. Analyse each graph and decide whether there is an intent to mislead the reader or if the information is presented honestly.

Check your answers by turning to the Appendix.

Conclusion

You've probably heard the old adage, believe half of what you see and none of what you hear. This is just a warning that things are not always as they seem. You have certainly become more aware of this as you completed this section.

In this section you looked at the strengths, weaknesses, and biases of samples and data collection methods. You analysed ways in which statistical information is presented, and looked for information that is missing so that an unbiased opinion could be formed.

You should now be ready to read and interpret statistical data presented in various media sources with a critical eye and an open mind.

Assignment

Assignment Booklet

You are now ready to complete the assignment for Section 2.

Section 3: Probability and Decision Making

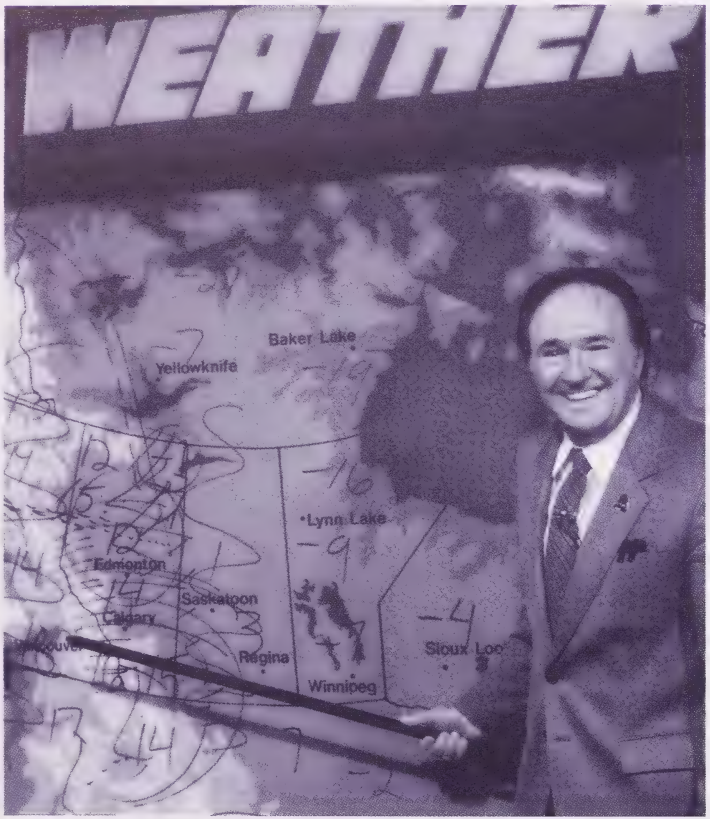


PHOTO SEARCH LTD.

Meteorologists use statistics and probability in creating and reporting their weather forecasts. When you see a weather forecast predicting a 40% chance of rain for the weekend, what does this mean to you? Would you go camping or plan for a round of golf for the weekend?

Other instances in which people use probability in their decision making may be whether to buy a lottery ticket or how to choose what numbers to play. Many participants may have difficulty understanding the probabilities involved in these activities and how much of their decision is based on the theoretical probability and how much is based on personal judgement.

In this section you will look at how people base decisions and future directions upon the probability of certain events happening. You will discover how to calculate the probability of independent events and use this knowledge in solving related problems.

Activity 1: Problems Involving Independent Events

Have you ever played Monopoly™? In the game of monopoly, when you land in jail, you have to miss two turns or roll doubles in order to move around the board again.

What is the probability of rolling doubles? What is the probability of rolling doubles two times in a row?

In previous mathematics courses you may have used a tree diagram or you may have done an experiment in order to determine the probability of rolling doubles. You used the following formula to calculate the theoretical probability.

$$P(event) = \frac{\text{Number of Favourable Outcomes}}{\text{Number of Possible Outcomes}}$$

You may have shown all the ways two dice could land as follows.

1, 1	2, 1	3, 1	4, 1	5, 1	6, 1
1, 2	2, 2	3, 2	4, 2	5, 2	6, 2
1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
1, 4	2, 4	3, 4	4, 4	5, 4	6, 4
1, 5	2, 5	3, 5	4, 5	5, 5	6, 5
1, 6	2, 6	3, 6	4, 6	5, 6	6, 6

In this activity you will use a mathematical statement to define the probability of independent events.

To find the probability of rolling doubles, you can count the number of doubles possible and the total number of combinations possible. In this case you have six doubles and 36 possible combinations.

The probability of rolling doubles the first time is as follows:

$$P(D_{first}) = \frac{6}{36} \text{ or } \frac{1}{6}$$

The probability of rolling doubles the second time is as follows:

$$P(D_{second}) = \frac{6}{36} \text{ or } \frac{1}{6}$$

The probability of rolling doubles twice in a row is the product of the two individual probabilities. It is calculated as follows:

$$P(D_{first,\ second}) = \frac{1}{6} \times \frac{1}{6}$$
$$= \frac{1}{36}$$

To find the probability of several independent events happening, multiply the probabilities of each of the events.

Example 1

Calculate the probability of rolling 2 on a die and getting heads when flipping a coin.

Solution

The probability of rolling a two on a die is $P(2) = \frac{1}{6}$.

The probability of getting heads on a coin is $P(H) = \frac{1}{2}$.

Therefore, the probability of rolling a two on a die and getting heads on a coin is as follows:

$$P(2 \text{ and } H) = \frac{1}{6} \times \frac{1}{2}$$
$$= \frac{1}{12}$$

Example 2

Calculate the probability of not getting a 1 or a 2 when rolling two dice.

Solution

The probability of getting a 3, 4, 5, or 6 on each die is $P(3,\ 4,\ 5,\ or\ 6) = \frac{4}{6}$.

Thus, the probability of getting a 3, 4, 5, or 6 on both dice is as follows:

$$P(3,\ 4,\ 5,\ or\ 6) = \frac{4}{6} \times \frac{4}{6}$$
$$= \frac{16}{36}$$
$$= \frac{4}{9}$$

The probability of not getting a 1 or 2 when rolling two dice is $\frac{4}{9}$.

There are numerous problems involving probability of independent events which can be solved.

Example 3

Brad creates a five-digit password for his computer voice mail. Calculate the probability that someone else can get into his voice mail by randomly selecting the five digits.

Solution

The probability of each digit being selected is $\frac{1}{10}$ (using 0 as a possible digit).

Thus, the probability of the five digits being selected is as follows:

$$P(\textit{password}) = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$$
$$= \frac{1}{100\,000}$$

The probability that someone will get into Brad's voice mail by randomly selecting digits is $\frac{1}{100\,000}$.

Example 4

Erica tosses three quarters. Calculate the probability that they will all land heads.

Solution

The probability of any one coin coming up heads is $\frac{1}{2}$.

Thus, the probability of all three coins coming up heads is as follows:

$$P(3H) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{8}$$

The probability of all three quarters coming up heads is $\frac{1}{8}$.

Now solve the following problems on probability of independent events.

1. In the game of backgammon, players roll a pair of dice to determine each move. If you roll doubles, you can double the number of moves you make. Calculate the probability that you can roll two 1s followed by two 6s.

2. Minal tosses a dime, a nickel, and a quarter at the same time.

   a. Calculate the probability of all three coins coming up tails.

   b. Calculate the probability of getting heads on the dime, tails on the nickel, and tails on the quarter.

   c. Compare your answer to question 2.a. with your answer to question 2.b. What does this suggest about the probability of each event?

   d. What other events are possible? Is the probability of these events occuring also $\frac{1}{8}$?

3. Lawrence has been flipping a penny. The penny has come up heads on six tosses in a row. He thinks it will come up heads on the next toss since it has come up heads so many times in a row. His friend Heather says it should come up tails since tails has not come up for so long.

   a. Who do you think is correct? Explain.

   b. What is the probability of heads on the next toss?

4. Sonia uses three single digits for her combination lock. What is the probability of someone guessing her combination by randomly selecting digits?

5. Sharon and Gertek were playing Monopoly™. Sharon said, "Let's change the rules. Let's use two dice. For every roll of the dice you move one space forward if a 3 or 4 comes up. Otherwise, you move two spaces backward." Will Sharon or Gertek move forward or backward after nine tosses?

6. Set up an experiment to check your prediction in question 5.

   a. Are you moving forward or backward after 9 tosses?

   b. How does the experimental probability for a 3 or 4 coming up when tossing two dice compare to the theoretical probability calculated in the answer to question 5?

7. You have a bag with three red, three orange, and three purple candies.

   a. What is the probability of drawing a purple candy?

   b. How many candies would you have to draw before you can be certain to draw a purple one?

8. You toss three quarters at the same time.

   a. Calculate the probability of getting three heads.

   b. Calculate the probability of getting a head and two tails.

9. A weather report indicates there is a 60% chance of rain on Monday, a 50% chance of rain on Tuesday, and a 30% chance of rain on Wednesday. Calculate the probability it will rain on all three days.

10. Mary is a basketball player who has a 70% record for sinking foul shots.

    a. Calculate the probability she will make two shots in a row.

    b. Calculate the probability that she misses both foul shots.

    c. Calculate the probability that she will make the first shot and miss the second.

Check your answers by turning to the Appendix.

In games of chance people often feel that certain numbers have a better chance of coming up than others. They are using intuition or their own perception to make the decision. This is called **subjective judgement**.

You can use the following experiment to compare theoretical probability, experimental probability, and subjective judgement regarding tossing a pair of dice.

11. Roll two ordinary dice together 100 times and record the number of times each sum occurs in a chart like the following.

Sum	Tally	Occurrences
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

Remember:

$$P(\text{event}) = \frac{\text{Number of Favourable Outcomes}}{\text{Number of Possible Outcomes}}$$

12. Use your data from question 11 to find the experimental probability of getting the following sums. Express your answers in decimal form.

a. 2 **b.** 6 **c.** 7 **d.** 10

Since there are two different dice in question 12, the theoretical probability of getting a sum of 4 is $\frac{3}{36}$ or $\frac{1}{12}$. This is because you can get a sum of 4 in three different ways (3 and 1, 1 and 3, and 2 and 2), and there are a total of 36 different combinations you can get with the two dice.

13. **a.** What is the theoretical probability of getting a sum of 2? Explain.

b. How does this compare with the experimental probability?

14. **a.** Which sum is likely to appear most often? Explain.

b. Did the predicted sum appear most often?

15. **a.** Which sum is likely to appear least often? Explain.

b. Did the predicted sum appear least often?

16. Why might the theoretical and experimental probabilities not be the same?

17. When you toss a pair of dice, do you try concentrating on a particular number or pair of numbers and hope they come up? Do you think that certain numbers come up more often than predicted by theoretical calculations?

Check your answers by turning to the Appendix.

When you think that certain numbers will come up more often than predicted by theoretical calculations, you are using subjective judgement.

Now Try This

If you have access to a computer spreadsheet, do the following computer simulation and answer questions 18 to 21.

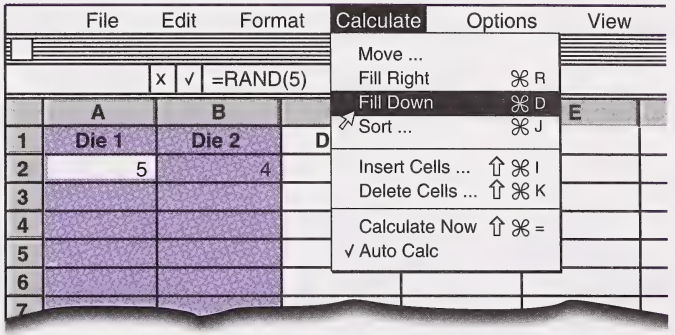
The following steps are for a *ClarisWorks*™ spreadsheet. Adapt the instructions if you have a different spreadsheet.

Step 1: Open a new spreadsheet file. Type Die 1, Die 2, and Die 1 + Die 2 in cells A1, B1, and C1 respectively.

Step 2: Enter =RAND(6) in cells A2 and B2, and enter =A2+B2 in cell C2.

Step 3: Highlight cell A2 and shift-click cell C100 (press shift and click on cell C100).

Step 4: Click on the Calculate menu, and select Fill Down.



Your spreadsheet should look like the following with random numbers between 1 and 6 in columns A and B and the sums in column C.

	A	B	C
1	Die 1	Die 2	Die 1+Die 2
2	5	4	9
3	5	2	7
4	4	2	6
5	4	5	9
6	1	3	4
7	3	3	6
8	6	4	10
9	4	5	9
10	2	2	4
11	5	1	6
12	5	2	7
13	5	3	8
14	4	3	7
15	3	1	4
16			

18. Create a table like the one in question 11 on your spreadsheet to record the number of times each sum in column C occurs. Count the occurrences of each sum and record them in the table.

19. What is the experimental probability of getting each of these sums?

a. 2 **b.** 6 **c.** 7 **d.** 10

20. How do these results compare with your answers to question 12?

21. Repeat the program another time and add the tallies for the new sums to the table. Are the experimental probabilities getting closer to the theoretical calculations?

22. Use the x-y line graph in Make Chart to create a graph of occurrences versus sums when tossing two dice.

23. Draw a smooth curve through the middle of the points. What shape is the curve?

Check your answers by turning to the Appendix.

You will encounter the bell-shaped curve in many future mathematics courses involving statistics.

Use a computer spreadsheet program to simulate a "toss" of three coins. The following steps are for *ClarisWorks*™.

Step 1: Open a new spreadsheet.

Step 2: In cells A1, B1, and C1 type Coin 1, Coin 2, and Coin 3 respectively.

Step 3: In cells A2, B2, and C2 type =RAND(2). Let 1 represent heads and 2 represent tails.

Step 4: Highlight cell A2 and shift-click on cell C21.

Step 5: Click on the Calculate menu and select Fill Down to get twenty "tosses" of the three coins.

Step 6: Copy and complete a table like the following.

Event	Tally	Frequency
three heads		
one head, two tails		
one tail, two heads		
three tails		

24. What should you see in any one row to represent the following?

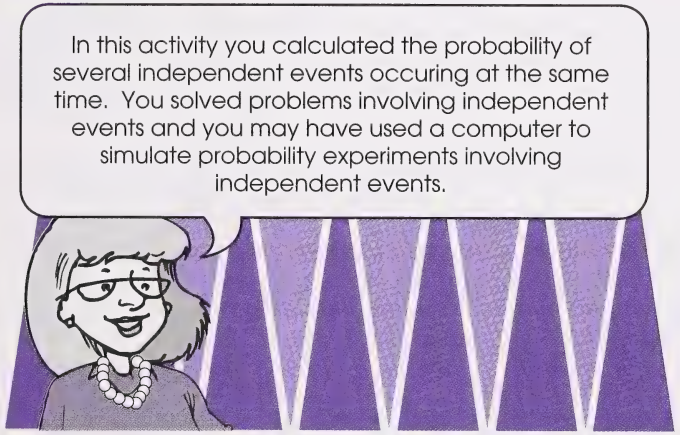
a. 3 heads

b. 1 head, 2 tails

c. 1 tail, 2 heads

d. 3 tails

Check your answers by turning to the Appendix.



Activity 2: Making Decisions Using Probability

The chance of rain for tomorrow is 30%. What does this statement mean to you? Is it quite likely or not too likely that it will rain tomorrow?

The probability of drawing a spade from a deck of 52 cards is $\frac{13}{52}$ or $\frac{1}{4}$. Is this quite likely to happen? What about the probability of drawing an ace of spades?

What is the probability that the Toronto Blue Jays will win the World Series this year? This probability is more difficult to predict and impossible to calculate because of the number of variables involved.

Of the previous three examples of probability, only the one involving choosing cards from a deck can be calculated accurately. The other two are dependent on numerous changing variables and cannot be calculated with complete accuracy.

In this activity you will analyse how probabilities are used to make decisions such as what numbers people pick for a lottery draw, or whether to go camping this weekend after listening to the weather forecast.

Decisions based on probability may be the result of a combination of theoretical calculations, experimental results, and subjective judgements. Often, however, when faced with a decision that has to be made quickly, it may be based more on a subjective judgement than a calculated probability.

Do you know anyone who plays Lotto 6/49 or Super Lotto?

1. Ask one or two people who play Lotto 6/49 or Super Lotto how they pick their numbers. Record their responses.

2. Sandra picks the numbers that come up most often. Do these numbers have a greater probability of coming up?

3. There are 13 983 816 possible combinations of numbers in Lotto 6/49.

   a. If you buy one ticket, what is the probability you will win the six-number prize?

   b. If you buy 10 tickets, what is the probability you will win the six-number prize?

   c. If you bought 13 983 816 tickets, all with different six-number combinations, could you say that you would be certain to win the Lotto 6/49? Would there be other winners?

4. People often get together in groups to buy lottery tickets. If there are twenty people in the group, they buy 20 lottery tickets. What is the probability for each person of winning part of the six-number prize? Round your answer to the nearest 100 000.

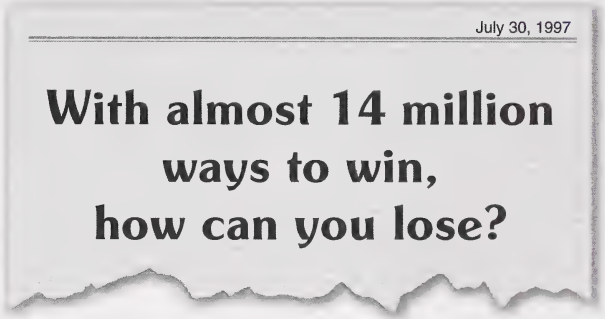
5. How many tickets would you need to increase your probability of winning to about 1 in 14 000?

6. Do you think that certain numbers have a greater probability of coming up than other numbers?

7. Why is it possible for there to be more than one winner of the six-number prize?

8. The probability of winning $10 is about 1 in 57. If you bought 57 tickets, would you be guaranteed to win $10?

The following lottery advertisement appeared in a magazine.



9. How would you rewrite this statement to reflect the actual probability of winning? Explain your answer.

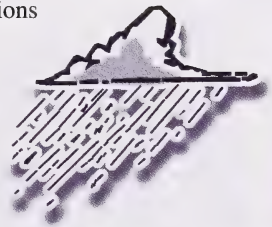
Check your answers by turning to the Appendix.

Search the Internet for information about Lotto 6/49. Find the probability for winning each prize.

10. Why is the probability of winning any prize better than the probability of winning any one prize?

Check your answer by turning to the Appendix.

Lotteries are not the only area where decisions are based on probability. The weather forecast for today is 40% **chance** of rain. What does a 40% chance of an event mean? A 40% chance of an event occurring is the probability of the event occurring expressed as a percent. A 40% chance is the same as a probability of 0.4. A 40% chance of rain also means that given the predicted conditions rain has fallen 40 times out of 100 in the past.



11. René wants to go camping this weekend with some friends. He hears that the forecast is for a 60% chance of rain in the area they want to go camping.

a. What is meant by a 60% chance of rain?

b. If René goes camping, is it certain that it will rain for part of the weekend?

c. What criteria might René use to decide whether or not to go camping?

12. Obtain the three- or five-day forecast from the weather page in a daily newspaper or from a television or radio weather report. Follow the predictions for the forecast period.

a. Graph the predicted and actual temperatures. How accurate were the predictions?

b. Compare the predicted and actual precipitation amounts. How accurate were the precipitation predictions?

Check your answers by turning to the Appendix.

Society uses probability in marketing, opinion polls, and weather forecasting.

Read the following article, that appeared in an issue of *The Edmonton Journal*, and answer the questions that follow.

The tale of 20 wallets

Over the last few weeks we dropped wallets containing $30 in cash, as well as identification, in 20 different Edmonton locations, to see what would happen. Here are the results:

#1
Place: Edmonton Centre, food court, pay phone
Finder: woman takes wallet to security guard
Outcome: wallet returned, money missing
Conclusion: someone took a finder's fee

#2
Place: U of A transit zone, bus bench
Finder: Jennifer Morton
Outcome: hops LRT to return wallet immediately
Conclusion: 'Honest Jen' deserves medal

#3
Place: Whyte Ave. and 103 St., on newspaper box
Finder: 20-something anonymous woman
Outcome: wallet, missing in action
Conclusion: don't leave wallet on red newspaper boxes

#4
Place: Liquor World, 137th Ave. and 93rd St.
Finder: nice, but anonymous lady
Outcome: turns wallet in, store staff return it
Conclusion: a toast to all involved

#5
Place: swanky High Street sidewalk
Finders: Debbie Newton, Joanne Bailey
Outcome: calls us immediately on cell phone
Conclusion: even saints have cell phones

#6
Place: an ATM in WEM, near Bourbon Street
Finders: an anonymous couple
Outcome: the wallet is long gone
Conclusion: guess they needed cash

#7
Place: on the road, inside neighborhood of Promontory Point
Finder: Jose Vega
Outcome: called immediately, wallet returned
Conclusion: this wallet wasn't road kill after all

#8
Place: spiral walk/bike ramp under LRT bridge

Finders: two trendy-looking young women
Outcome: dropped anonymously at *The Journal*, $5 missing
Conclusion: maybe they needed $5 for bus fare

#9
Place: The Drag, at 96th St. and 103rd Ave.
Finder: 50-something man in jeans and leather jacket
Outcome: that wallet is going, going . . . it's gone
Conclusion: it's almost too cliche to comment on

#10
Place: downtown Greyhound bus depot
Finder: Larry Jackson, ticket agent
Outcome: wallet back, intact
Conclusion: bus people are doggedly honest

#11
Place: Grant MacEwan college, downtown
Finder: student Aileen Cameron.
Outcome: turned in and returned intact
Conclusion: we could learn from this student

#12
Place: Baccarat Casino, near VLT machines
Finder: woman dressed in bright colors
Outcome: wallet is pocketed and kept
Conclusion: perhaps she changed her luck

#13
Place: #11 bus to Belgravia
Finder: James Franks
Outcome: turns it into ETS driver
Conclusion: there's something about bus people

#14
Place: bus bench on 100 Ave, south of *The Journal*
Finder: Louis Grimble
Outcome: Grimble brings it back to us
Conclusion: we'll call him Saint Louis

#15
Place: Legislature, fourth floor, men's washroom
Finder: Jerry Burke, provincial worker
Outcome: wallet's back, intact
Conclusion: give this guy a wage roll-up

#16
Place: city hall, couch, second floor
Finder: anonymous citizen
Outcome: wallet turned in, intact
Conclusion: thanks, whoever you are

#17
Place: J. Percy Page High School, near bikes
Finder: Cliff Oatway, unemployed
Outcome: Cliff drives the wallet to *The Journal*
Conclusion: someone should hire this guy

#18
Place: sidewalk outside Westmount Jr. High
Finder: Michael Brightwell, grade 7
Outcome: turns wallet into school office
Conclusion: kids these days . . . are just fine

#19
Place: field near Capilano Elementary School
Finder: an anonymous bike-riding kid
Outcome: no calls, no letters, no wallet
Conclusion: but some kids do the darndest things

#20
Place: Northlands race track, The Spectrum
Finders: an anonymous couple
Outcome: they literally run to turn it in
Conclusion: what were the odds? In Edmonton? Pretty good.

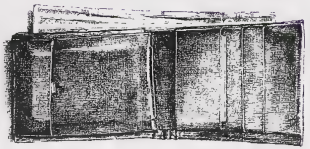
[1]

13. What probability question is the article attempting to answer?

14. How many wallets with $30 cash were dropped? How many wallets and cash were returned? What fraction of wallets and cash were returned?

[1] "The Tale of 20 Wallets," *The Edmonton Journal*, 18 May 1997, G3. Reprinted by permission.

15. What conclusion does the article reach regarding lost wallets in the city of Edmonton? Do you agree with this conclusion?



16. Where were most of the wallets dropped? Why?

17. Is there any bias in how this experiment was conducted?

18. Can the conclusion be made about the entire city of Edmonton?

19. Find two examples from newspapers, magazines, radio, or television that use probability to market a product or service, or to answer an opinion poll. Answer the following questions for each example.

    a. How is probability used in the example?

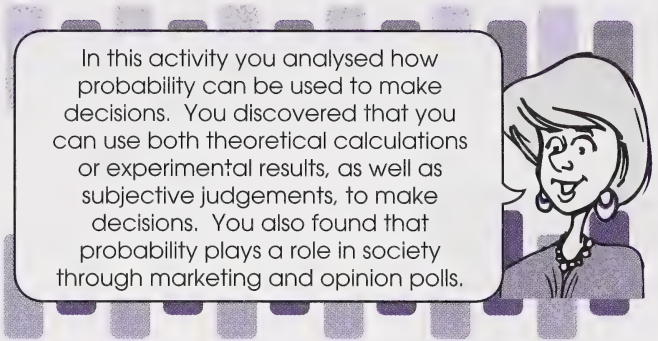
    b. Is the data valid?

    c. Is the data presented in an honest way, or is it misleading?

    d. What assumptions are made?

    e. What conclusions (if any) are drawn?

Check your answers by turning to the Appendix.



Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may do both.

Extra Help

The probability of independent events is found by multiplying the probabilities of each individual event.

1. A spinner has four colours (red, green, blue, and yellow). Calculate the probability of spinning blue and rolling a number less than 3 on a die.

2. What is the probability of getting heads on a coin toss, rolling an even number on a die, and drawing an ace of spades from an ordinary deck of 52 playing cards?

Check your answers by turning to the Appendix.

Making decisions involving probabilities can involve theoretical calculations, experimental results, and subjective judgement. You make a theoretical calculation when you find that the probability of winning Lotto 6/49 are about one in 1 398 382 if you buy 10 tickets. You make a subjective judgement when you decide to buy 10 Lotto 6/49 tickets because with 10 tickets you have 10 chances to win.

3. State whether each statement represents a theoretical calculation, an experimental result, or a subjective judgement.

   a. Sally tosses three coins 40 times, records the results, and finds the probability of three heads to be $\frac{6}{40}$ or $\frac{3}{20}$.

   b. Arron hears there is a 50% chance of rain for the weekend. He decides not to go camping since he feels there is a good chance he will get wet.

   c. Allison calculates the probability of getting a 6 on a die and heads on a toss of a quarter to be $\frac{1}{6} \times \frac{1}{2}$ or $\frac{1}{12}$.

   d. Eric decides he wants to sit at the back of the plane on his holiday flight because he has heard that the chances of surviving a plane crash are 10% better if you are in the tail section of the plane.

Check your answers by turning to the Appendix.

Enrichment

Expected Winnings

There are many different provincial lotteries, as well as various organizations that run lotteries to raise money for a specific event. The number of tickets sold varies from lottery to lottery.

If you were to buy a lottery ticket, how much would you expect to win? The expected winnings is the amount a single ticket could win if all tickets received part of the prize.

To calculate expected winnings, divide the total prize money by the number of tickets sold.

Example

Suppose an organization sells 40 000 tickets at $10 each, and offers prizes as follows.

1	$100 000 prize
2	$10 000 prizes
20	$1 000 prizes
200	$100 prizes

What amount can you expect to win if you buy a ticket?

Solution

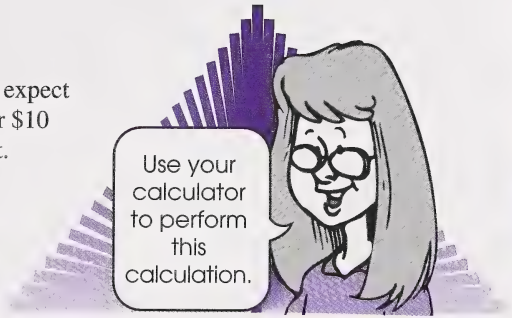
$$\frac{(1\times 100\,000)+(2\times 10\,000)+(20\times 1000)+(200\times 100)}{40\,000}$$

$$=\frac{100\,000+20\,000+20\,000+20\,000}{40\,000}$$

$$=\frac{160\,000}{40\,000}$$

$$=\$4$$

Therefore, you can expect to win $4 with your $10 purchase of a ticket.



1. Use the information from Example 1 to answer the following:

   a. Is the $10 ticket purchase a wise investment based on the expected winnings?

   b. Is this the amount anyone will actually win?

   c. If the expected return is only $4 on a $10 investment, why do people buy lottery tickets?

2. Calculate the expected winnings for the following.

   a. 10 000 tickets sold at $2 each with the following prizes.

1	$3000 prize
5	$1000 prizes
10	$100 prizes

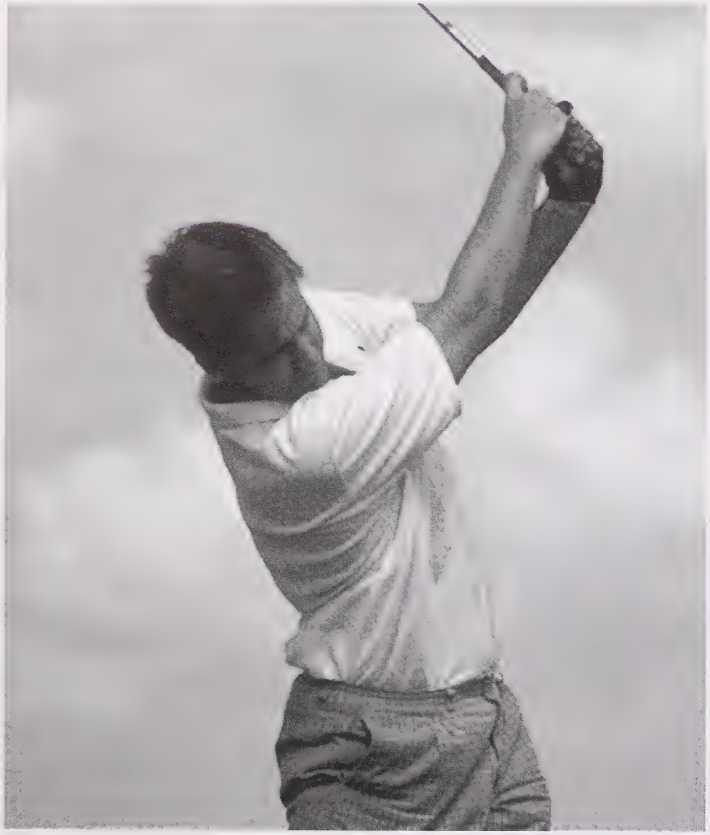
   b. 1500 tickets sold at $5 each with the following prizes.

1	$2500 prize
1	$1000 prize
5	$100 prizes

3. Calculate the probability of winning any prize in each of questions 2.a. and 2.b. Which lottery gives the better chance of winning a prize?

Check your answers by turning to the Appendix.

Conclusion



In this section you analysed how people interpret events involving probability. Do they use theoretical calculations or experimental results to make their decisions, or do they rely on their own subjective judgements?

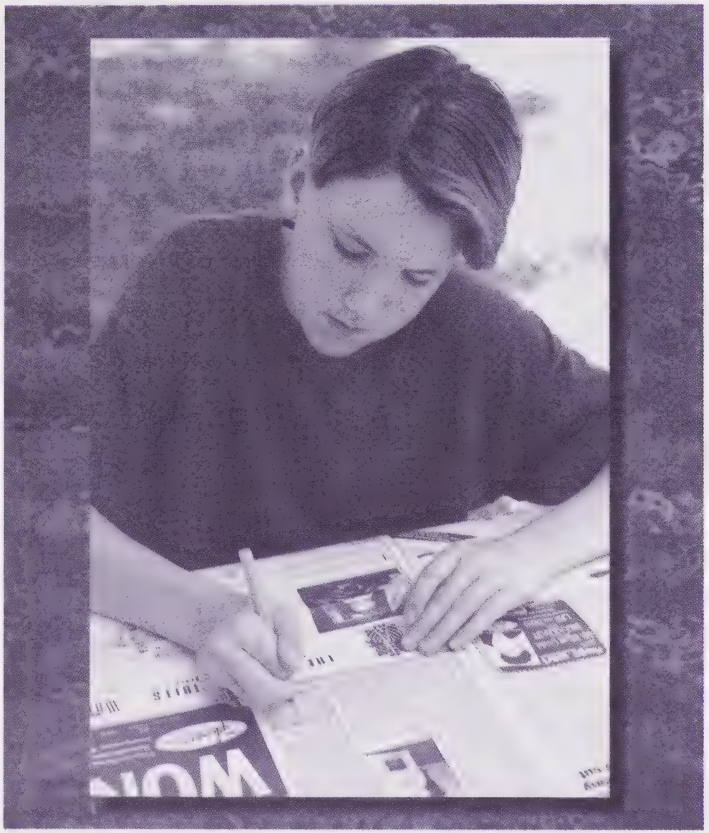
You explored the probability of independent events, and you had the opportunity to simulate some events involving the probability of independent events.

Do you now realize that some of your decisions are based on subjective judgement as well as some understanding of the theoretical calculation of the probability of an event? If the weather report states that there is a 60% chance of rain tomorrow and you have plans to go golfing, you may find yourself taking a chance and going anyway.

Assignment

You are now ready to complete the assignment for Section 3.

Module Summary



In this module you discovered the meaning of bivariate data and designed some experiments involving bivariate data. You then analysed that data using scatter plots and approximated the position of a line of best fit. In Section 2 you investigated how statistics are used or misused by individuals and groups. You analysed some examples of data from newspapers, magazines, radio, and television. In Section 3 you analysed how people base their decisions involving probability partly on theoretical calculations and partly on subjective judgement. You solved problems involving the probability of independent events and found that a computer could be a valuable asset in the study of probabilities.

You should now be ready to make wise decisions involving statistical data, predictions in games of chance, and opinion polls.

Final Module Assignment

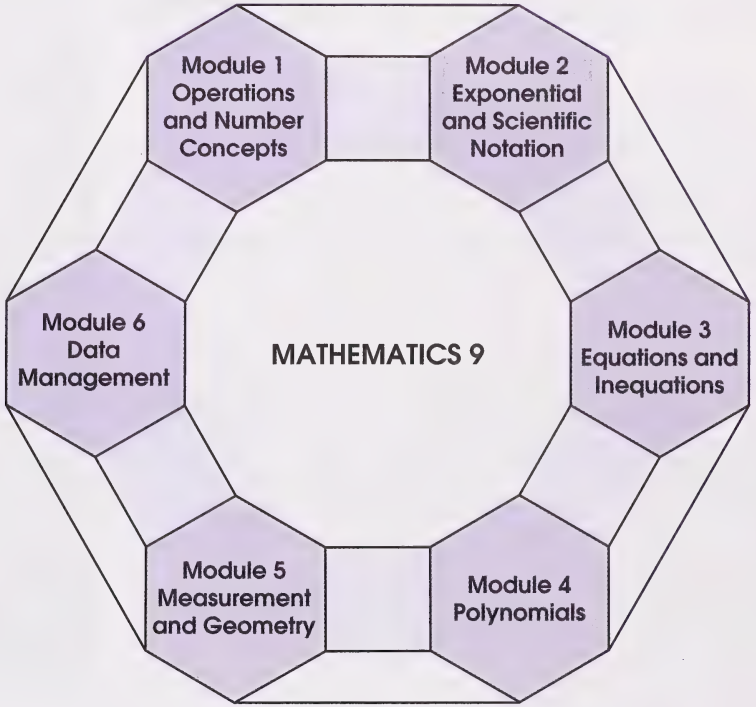
Assignment Booklet

You are now ready to complete the final module assignment.

Course Summary

JIM WHITMER PHOTOGRAPHY

Congratulations on completing the six modules of Mathematics 9. Hopefully you found the course interesting, informative, and challenging.



By now all your assignments should have been completed and reviewed. Now it's time for the last step in the Mathematics 9 course—the Final Test. With all the hard work and time you put into the course, you are sure to do well.

COURSE SURVEY FOR MATHEMATICS 9

After you have completed the assignments in this course, please fill out this questionnaire and mail it to the address given on the last page. This course is designed in a new distance learning format, so we are interested in your responses. Your constructive comments will be greatly appreciated, as future course revisions can then incorporate any necessary improvements.

Name _____ Age ❑ under 19 ❑ 19 to 40 ❑ over 40

Address _____ File No. _____

_____ Date _____

_____

Design

1. This course contains a series of Student Module Booklets. Do you like the idea of separate booklets?

_____

_____

2. Have you ever enrolled in a correspondence or distance learning course that arrived as one large volume?

❑ Yes ❑ No If yes, which style do you prefer?

_____

_____

3. The Student Module Booklets contain a variety of self-assessed activities. Did you find it helpful to be able to check your work and have immediate feedback?

❑ Yes ❑ No If yes, explain.

_____

_____

4. Were the questions and directions easy to understand?

❑ Yes ❑ No If no, explain.

_____

_____

5. Each section contains follow-up activities. Which type of follow-up activity did you choose?

   ❑ mainly extra help
   ❑ mainly enrichment
   ❑ a variety
   ❑ none

   Did you find these activities beneficial?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

6. Did you understand what was expected in the Assignment Booklets?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

7. The course materials were designed to be completed by students working independently at a distance. Were you always aware of what you had to do?

   ❑ Yes ❑ No If no, provide details.

   _____

   _____

8. This distance learning course may include an assortment of drawings, photographs, and charts.

   a. Did you find the visuals in this course helpful?

      ❑ Yes ❑ No Comment on the lines below.

      _____

      _____

   b. Did you find the variety of visuals in this course motivating?

      ❑ Yes ❑ No Comment on the lines below.

      _____

      _____

9. Suggestions for audiocassette, videocassette, and computer activities may have been included in the course. Did you complete these media activities?

   ❑ Yes ❑ No Comment on the lines below.

   _____

   _____

Only students enrolled in a Junior High course need to complete the following question.

10. The Student Module Booklet may have directed you to work with your learning facilitator. How well did you work as a team?

    Student's comments: _____

    _____

    _____

    _____

    Learning Facilitator's comments: _____

    _____

    _____

    _____

Course Content

1. Was enough detailed information provided to help you learn the expected skills and ojectives?

   ❑ Yes ❑ No Comment on the lines below.

   _____

   _____

2. Did you find the work load reasonable?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

3. Did you have any difficulty with the reading level?

❑ Yes ❑ No Please comment.

_____

_____

4. How would you assess your general reading level?

❑ poor reader ❑ average reader ❑ good reader

5. Was the material presented clearly and with sufficient depth?

❑ Yes ❑ No If no, explain.

_____

_____

General

1. What did you like least about the course?

_____

_____

2. What did you like most about the course?

_____

_____

Additional Comments

_____

_____

_____

_____

_____

_____

_____

Only students enrolled with the Alberta Distance Learning Centre need to complete the remaining questions.

1. Did you contact the Alberta Distance Learning Centre for help or information while doing your course?

   ❑ Yes ❑ No If yes, approximately how many times? _____

   Did you find the staff helpful?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

2. Were you able to fax any of your assignment response pages?

   ❑ Yes ❑ No If yes, comment on the value of being able to do this.

   _____

   _____

3. If you mailed your assignment response pages, how long did it take for their return?

   _____

   _____

4. Was the feedback you received from your correspondence or distance learning teacher helpful?

   ❑ Yes ❑ No Please comment.

   _____

   _____

   _____

   _____

Thanks for taking the time to complete this questionnaire. Your feedback is important to us. Please return this questionnaire to the address on the right.

If you are enrolled at the Alberta Distance Learning Centre and have been mailing your Assignment Booklets to ADLC, you may return this questionnaire with the final Assignment Booklet in the course.

Instructional Design and Development
Learning Technologies Branch
Box 4000
Barrhead, Alberta
T7N 1P4

APPENDIX

Glossary

Suggested Answers

Glossary

Bivariate data: data that involves a relationship between two measures

Census: all items or individuals of the population are used to collect data

Chance: a probability expressed as a percent

Correlation: the relationship, or lack there of, that exists between two variables

Data: factual information that is gathered and used for calculations, comparisons, and discussion

Dependent event: an outcome or event whose value or probability depends on another outcome or event

Extrapolate: to use information on a graph to estimate values that go beyond the graph

Independent events: outcomes or events whose value or probability does not depend on another outcome or event

For example, the probability of tossing six heads in a row with a coin is an independent event.

Interpolate: to read information directly from a graph

Line of best fit: a line drawn through the points of a scatter plot that best estimates the relationship between the two variables

Population: all of the individuals or items about which information is collected

Representative sample: a sample that has the same characteristics as the whole population being studied

Sample: a part of the population that is used to gather data about the whole

Scatter plot: a graph of a set of points representing the relationship between two sets of numbers or data

Subjective judgement: using intuition or one's own perception to influence a decision

Suggested Answers

Section 1: Activity 1

1. Answers will vary. Other examples could include the following:

- amount smoked and number of sick days taken per year
- amount of time studying and examination mark
- amount of fertilizer and crop yield
- height and size of feet

2. a. Ms. Coulson could give the students two types of marks on their work, assignments, projects, and tests. The first mark would measure their performance and the second mark would reflect the effort that the students put into their work. If Ms. Coulson collected this data throughout the year, she would have some basis for comparison.

b. Answers will vary. Some examples are given.

- How do you measure effort? (time, commitment, attentiveness,. . .)
- Should the students be aware they are being measured for effort? Will this influence results?
- Can Ms. Coulson be objective? Will a higher mark for effort be awarded due to a good performance mark?

3. a. You could set up an experiment to measure the height and the shoe size of people within various age groups (such as 5- and 6-year-olds, 15- and 16-year-olds, and 25-year-olds and over).

b. Answers may vary. Some factors which might influence the data are age, nationality, health, and weight.

4. Answers may vary. The following are some suggestions.

a.
- Is there a relationship between the amount of jogging (minutes per week) and longevity?
- Is there a relationship between the amount of jogging and the incidence of heart attacks?

b.
- Is there a relationship between the amount of smoking (packs per day) and illness (sick days per year)?
- Is there a relationship between the amount of smoking and longevity?

c.
- Does increasing the amount of fertilizer used increase the amount of produce yielded?
- Does the amount of fertilizer used affect the quality of the produce yielded?

d.
- Is there a relationship between the amount of time spent studying (hours per week) and the results on exams?
- Does the frequency of studying (times every week) affect the results on exams?

5. a. flu shot (yes/no) and number of bouts of flu per year

b. birth weight (in kilograms) and age at death

6. a. The data should be collected from mature people—that is, people who have reached their full adult height—in order to eliminate other factors. You should have used a sample of at least 30 people and then measured their height (in centimetres) and the circumference of their head (in centimetres). Your chart should look similar to the following.

Person	Height (cm)	Head Size (cm)
Jennifer	162	56.5
Robert	180	57.4
Travis	184	57.6
Rolanda	168	56.3

b. Your sample of about 30 people can range in age. Measure their weight (in kilograms) and their wrist size (in centimetres), and record the data in a chart similar to the following.

Person	Weight (kg)	Wrist Size (cm)
Wynonna	60	15
Matt	83	17.8
Alfred	88	18.1
Roberta	72	15.5

c. Measure, to the nearest tenth of a centimetre, the diameter and the circumference of a number of circular objects (such as a tin can, a basketball, and a roll of tape). Use a string and measure carefully. Organize the data in a chart similar to the following.

Object	Diameter (cm)	Circumference (cm)
tin can	7.5	22.8
basketball	25	79
roll of tape	12.2	37.8

7. a. The two variables are mathematics marks of students in a girls-only class and the marks of students in a regular integrated class. The relationship discussed is "Does being in a girls-only class affect the mathematics marks achieved by the girls?"

b. Mathematics scores are improved if girls are in a class without boys.

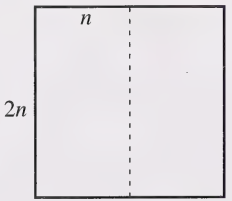
8. Answers will vary. Some examples of relationships that you could investigate are as follows:

- the price in Canadian dollars versus the price in U.S. dollars for books or magazines
- the extension of a spring versus the mass attached
- height versus span of hand from the tip of the outstretched thumb to the tip of the index finger
- temperature versus time of day over a two-day period

Your answer should include a chart similar to those in question 6 and a statement discussing the results of your experiment.

Now Try This

9. a. Draw a diagram.



Let n be the width of each new rectangle. Thus, $2n$ is the length of each new rectangle.

$$n+n+2n+2n=36$$
$$6n=36$$
$$\frac{6n}{6}=\frac{36}{6}$$
$$n=6$$

Now find the length of the rectangle.

$$2n=2(6)$$
$$=12$$

Since the length of the square is the same as the length of the new rectangle, the perimeter of the original square is $12\times 4=48$ cm.

b. $n+n+2n+2n=30$
$$6n=30$$
$$\frac{6n}{6}=\frac{30}{6}$$
$$n=5$$

Now find the length of the rectangle.

$$2n=2(5)$$
$$=10$$

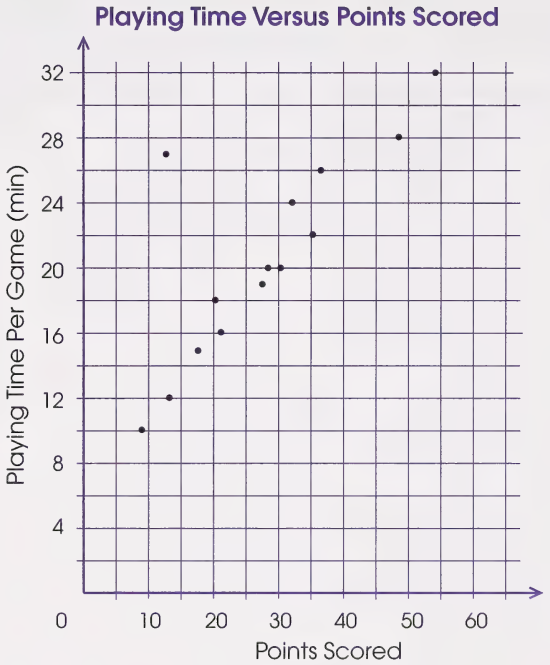
Therefore, the perimeter of the original square would be $10\times 4=40$ cm.

c. Yes, the perimeter of each new rectangle is three-quarters the perimeter of the original square. This is because you've effectively eliminated one side by folding the square in half.

Section 1: Activity 2

1. The fuel consumption increases as the mass of the vehicle increases.

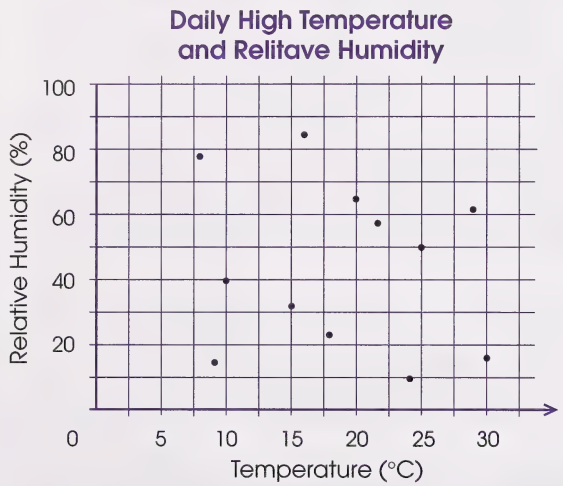
2. a.



b. When the amount of playing time increases, the number of points scored increases.

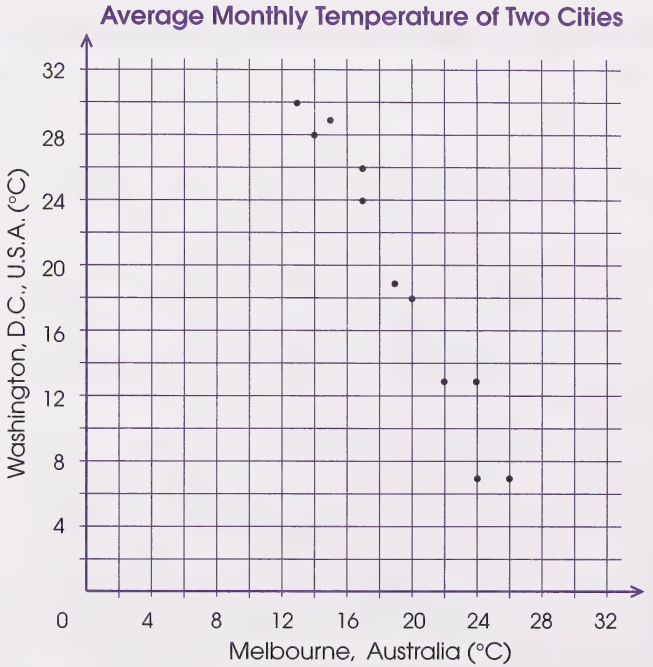
c. Yes, player #2 doesn't seem to fit. Perhaps the player plays defence and is noted for his or her defensive abilities and not scoring prowess.

3.



There doesn't seem to be any trend or noticeable relationship between temperature and relative humidity.

4.



As the temperature rises in Washington, D.C., it decreases in Melbourne and vice versa. This relationship occurs because their seasons are opposite one another. For instance, winter in Washington coincides with summer in Melbourne.

5. **a.** The points plotted will rise to the right.

b. The points plotted will rise to the right.

c. The points plotted will show no particular relationship.

d. The points plotted will rise to the right.

e. The points plotted will fall to the right.

Now Try This

6. a. Rule: To obtain any number in each row after Row 1, add the number directly above to the two numbers to its left.

Row 1: 2 2 2 ← 2 + 2 + 2 = 6

Row 2: 2 4 6 4 2

Note: Any position that has no number in it should be regarded as a zero.

Row 1: 2 2 2 ← 2 + 2 + 0 = 4

Row 2: 2 4 6 4 2

∴ Row 5: 2 10 30 60 90 102 90 60 30 10 2

b. Each row reads the same from left to right as it does from right to left.

Section 1: Activity 3

1. Yes, the line approximates the points fairly well.

2. The line suggests that the relationship is linear (points lie along a line). The line also shows that the points rise to the right.

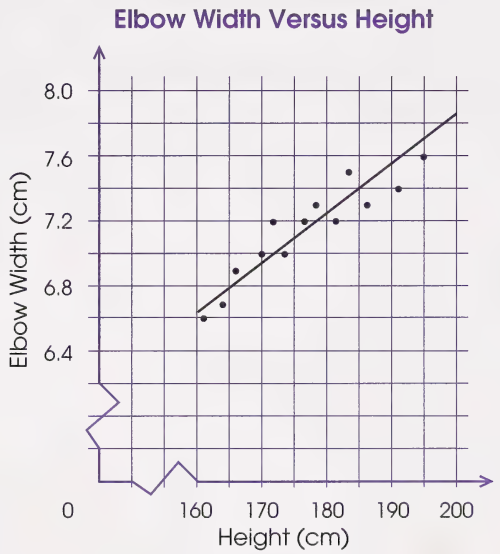
3. a. The fuel consumption of a vehicle with a mass of 1900 kg is about 13.5 L/100 km.

b. According to the scatter plot, the mass of a vehicle with a fuel consumption of 3.5 L/100 km would be about 500 kg.

c. Yes, the line crosses the axis at the coordinate $(0, 0)$. This is reasonable because when the mass of a vehicle is zero, the fuel consumption would be zero.

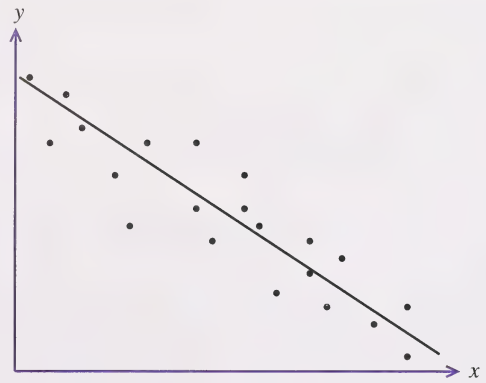
d. The line of best fit represents the average fuel consumption for a vehicle with a given mass. The vehicles represented by the points not on the line of best fit are more or less fuel efficient than the average vehicle.

4. Your line of best fit should look somewhat like the following.

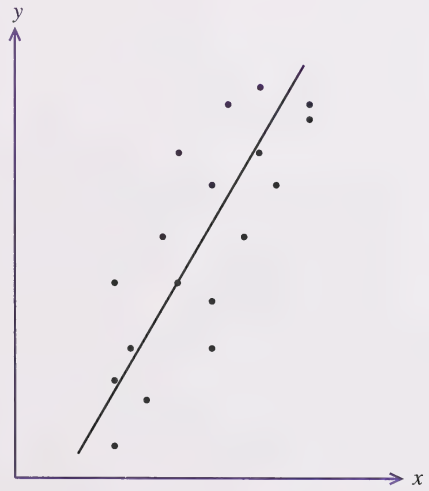


5. These lines of best fit are approximations. Your lines may not be exactly the same, but they should be close to the ones given.

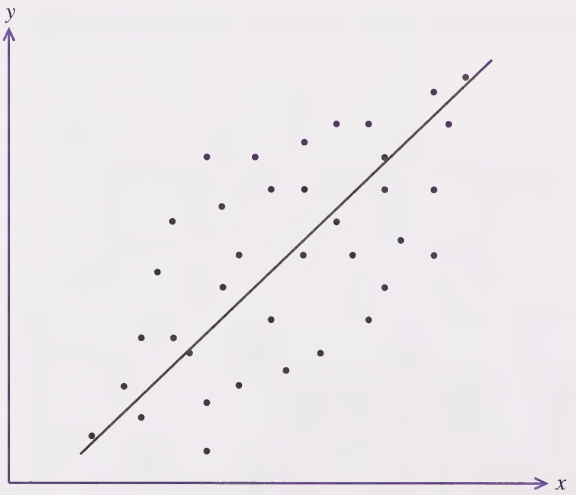
a.



b.

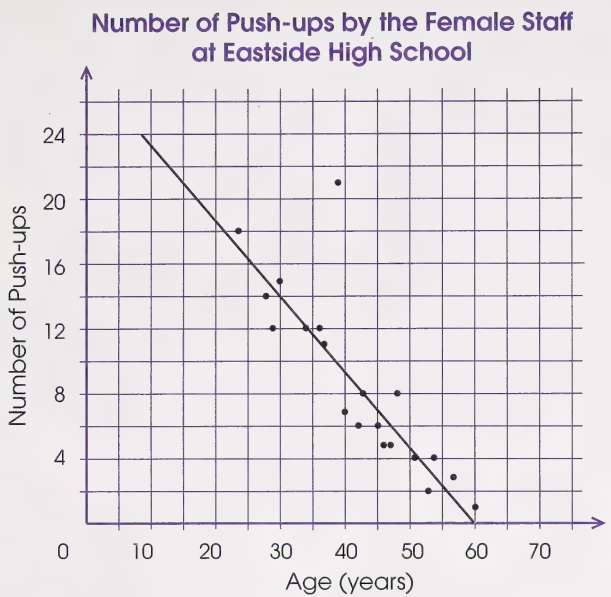


c.



6. You should have found it easiest to locate the line of best fit on the scatter plot in question 5.a., since the points are closest to being in a line.

7. a. Your scatter plot and line of best fit should look as follows.



b. As each of the female staff gets older, the number of push-ups they can do decreases.

c. The one exception could be a physical education teacher or someone who works out on a regular basis.

d. You should have ignored the one point that was an exception when drawing your line of best fit.

e. According to the graph, staff that are 60 years old and over cannot do any push-ups. This may not be true.

8. a. Scatter Plots A, B, C, and E show some kind of trend.

b. Scatter Plots A, B, and C appear to be linear.

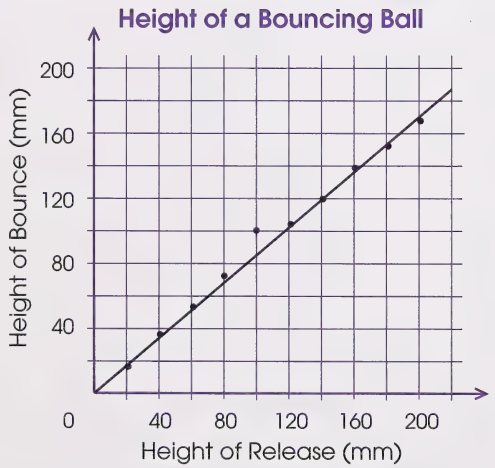
c. You would draw a smooth curve through the points in Scatter Plot E.

d.

The data has no apparent trend. There does not appear to be any relationship between the variables.

9. a. Answers will vary. A sample chart and scatter plot is given. Your chart and scatter plot should be similar.

Height of a Bouncing Ball			
Height of Release (mm)	Height of Bounce (mm)	Height of Release (mm)	Height of Bounce (mm)
20	17	120	103
40	37	140	120
60	52	160	138
80	72	180	152
100	90	200	168



b. The higher the ball is when dropped, the higher it bounces.

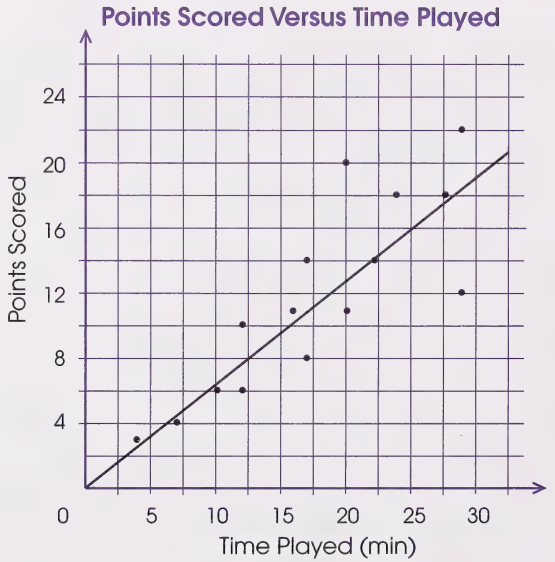
c. Some points may be further from the line of best fit due to experimental error.

d. Answers will vary depending on your experimental results. In this case, a ball dropped from 100 mm will bounce about 85 mm.

e. Yes, the type of ball should make a difference.

f. Answers will vary. You should have found that a hard ball, like a superball, will bounce the highest.

10. a.



b. Generally, as the playing time increases, so does the number of points scored.

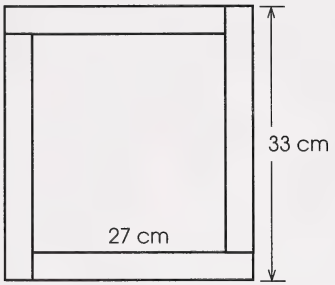
c. Is Ryning contributing in other ways to the team (rebounds, assists, blocked shots, and so on) or should her playing time be decreased? Perhaps House should be getting more playing time based on her scoring average.

d. About 16 points.

e. She might be able to score about 19 points.

Now Try This

11. Draw a diagram.



a. $A = s^2$

$= (27)^2$

$= 729$

The area of the inner square is $729\ \text{cm}^2$.

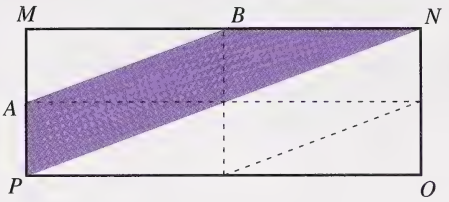
b. $A = s^2$

$= (33)^2$

$= 1089$

The area of the outer square is $1089\ \text{cm}^2$.

12. Find out the fraction of the rectangle that is not shaded first.



ΔMBA represents $\frac{1}{8}$ of the rectangle.

ΔNOP represents $\frac{4}{8}$ of the rectangle.

Therefore, $\frac{1}{8} + \frac{4}{8} = \frac{5}{8}$ of the rectangle is not shaded, and $\frac{3}{8}$ of the rectangle is shaded.

13. Answers will vary. Here are some examples of words that may have been found.

retain	elation	lapse
ropes	esprit	learn
reposit	earth	least
resist	enthral	latrine

atone
astern
aspen
antler
another

oriental
opera
opens
oaths
oasis

hostile
horse
hotel
honest

personal
piano
planet
postal

triple
throne
treason
their
tonsil

north
noise
nostril
niter
nosiest

hospital
hostel
hernia
heist

instep
inlet
inertia
inherit
inhale

sprain
stripe
shrine
slant
snare

plaster
praise
polish
prison

Section 1: Follow-up Activities

Extra Help

1. **a.** $y = 116$ **b.** $y = 107$ **c.** $y = 101$

2. **a.** 105 **b.** 150 **c.** 170

Enrichment

1. Answers will vary. Your report should include a scatter plot and a line of best fit.

a. The dots should approximately be in a straight line that rises to the right.

b. Any data from an experiment may vary due to experimental error.

c. Answers will depend on which investigation you completed.

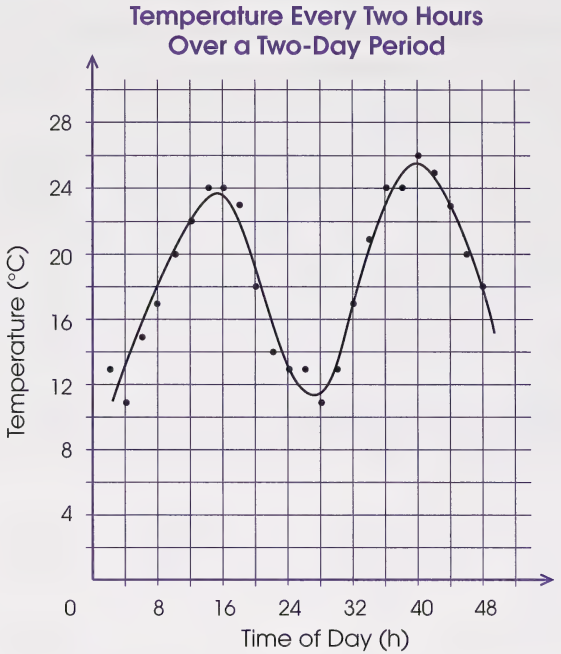
- A spring increases in length as the mass increases.
- The mass of a substance increases as its volume increases.
- The Canadian price for books and magazines increases as the American price increases.

d. Answers will depend on which investigation you completed. Use the line of best fit on your scatter plot to interpolate and extrapolate various additional values for the relationship.

2. Answers will vary due to season and location.

a. Your data and scatter plot may look something like the following.

Temperature Every Two Hours Over a Two-hour Period			
Time of Day (h)	Temperature (°C)	Time of Day (h)	Temperature (°C)
2	13	26	13
4	11	28	11
6	15	30	13
8	17	32	17
10	20	34	21
12	22	36	24
14	24	38	24
16	24	40	26
18	23	42	25
20	18	44	23
22	14	46	20
24	13	48	18



b. Refer to question 2.a. for the line of best fit. You should have drawn a smooth curve through the points.

c. The line of best fit is a curve, not a straight line as in question 1.

Section 2: Activity 1

1. According to the article, the data was collected by interviewing 890 Canadians.

2. The data was collected this way so the sample would be representative of all Canadians.

3. Yes, the interview method was appropriate for the type of data collected and for the issue. People may be hesitant to respond to a questionnaire on this issue.

4. Yes, the information is presented clearly and without any attempt to influence the reader.

5. The sample in the article is made up of working adults. You can tell who makes up the sample from the analysis of the responses (such as finances, work, and parenting).

6. Yes, the data collected and the resulting conclusions would most likely be typical of adult, working Canadians.

7. **a.** Census takers drop off questionnaires at every household in Canada before census day. They return after census day and collect the questionnaires. The results are entered into computers.

   b. Answers will vary, depending on the year researched. In 1991, the census cost over $250 000 000.

   c. The data is used to determine electoral boundaries, federal payments to the provinces, provincial payments to cities, and to formulate social and economic policies in planning for schools, hospitals, and transportation systems.

   d. Although the census is quite accurate, it is not 100% accurate. Some households are missed and not all homeless people are counted.

8. Answers will vary. Some suggested answers are outlined as follows:

a.

Advantages	Disadvantages
• You can see personal reactions and react to them. • It's possible to get responses from everybody. • The interview can be made more personal. • Questions can be answered directly.	• The demeanor of the interviewer can influence the answers. • Some people don't like personal contact—and may decline to be interviewed or not be truthful.

b.

Advantages	Disadvantages
• It is easy to carry out. • It can reach a large number of people quickly. • It is easier to get back to people a second time for follow-up.	• Some people will hang up on you. • It is not as personal. • It may be difficult to find people at home. • It gets only immediate reactions.

c.

Advantages	Disadvantages
• There is a written record of responses. • It is easy to sort and classify data. • A person can respond at his or her leisure—gives time to think about questions. • Each person gets the question in the same form.	• The return rate is often very low. • Nobody is there to respond to any questions about the questionnaire. • You may not understand a question on the questionnaire.

9. Answers will vary. Some suggestions are as follows:

a. • Many people are at work during the day.

• Many people do not want to be disturbed from their favourite past time in the evening.

b. • Some people would be reluctant to be interviewed, others would be the opposite—get only one type of person.

• Being on television might influence peoples' responses.

c. • If there are too many questions, some may go unanswered, or not as much thought would go into all the responses.

d. • Some people would prefer privacy, thus wouldn't like to be interviewed in public.

• Some people may be more relaxed in a familiar, comfortable setting (like their home).

e. • More people may be encouraged to come forward.
• Some people would likely be more forthright or truthful.

f. • The percentage of questionnaires completed should be higher if a prize is offered.

g. • A computer-generated phone interview could get fewer responses than one conducted by a person.

10. **a.** This is a random sampling if every student has a telephone number.

b. This is a random sampling.

c. This is not a random sampling since not every student has a chance of being selected.

d. This is a random sampling.

e. This is a random sampling if there are approximately the same number of students in each grade.

f. This is a random sampling, but the size of the sample is not big enough for a true reading of students opinions.

11. Yes, the data appears to be presented clearly and honestly.

There is a question of how rounding is used in the first statement. The statement says there is good news for Edmonton's jobless Friday with fresh statistics showing the city's unemployment rate dropped nearly a whole percentage point in April.

Two paragraphs later you read that the change is 0.7% $(7.9 - 7.2)$. The rounding adds on about 40% $(0.3 \div 0.7 \times 100)$. This is significant in the given statistic.

12. You would tend to feel that the change in the jobless rate for Edmonton in April is more positive than it actually is.

13. The jobless rate for the city of Edmonton is decreasing, while the national jobless rate is increasing.

14. Answers will vary according to the data you collected. Refer to the questions and answers for the two articles presented in this activity.

15. This information is not given. Perhaps only two or three people were used in the tests.

16. No. The information states that the amount of water absorbed by Thirsty paper towel is compared to some other brands. There may have been other brands that absorbed as well or better than Thirsty.

17. There is bias in that it appears that only data favourable to Thirsty's absorption power is presented.

18. The advertisement could have shown and discussed the actual data from the tests. There could have been data comparing several tests with Thirsty and several other brands.

19. No, most likely only a small number of dentists were consulted.

20. Not necessarily. Although samples are often used to represent an entire population, cases involving particular preference may not always be true.

21. Answers will vary depending on the article or advertisement picked. Refer to the analysis of the two previous advertisements.

Section 2: Activity 2

1. a. Yes, the graph presents the information accurately with $300 of sales in June and $600 in July.

b. It looks like sales in July are at least four times the sales in June, not twice as big as the figures accurately show.

c. You could depict the information more accurately if you used bars of equal width instead of the ice cream cones.

d. Tanya's purpose is to emphasize the increase in ice cream sales in July. Yes, she achieves her purpose quite effectively.

e. You may or may not wish to change it. The graph is very effective for the purpose for which it was designed.

2. a. The bottom portion of the graph is missing. This emphasizes the difference between the amount of sales between the companies.

b. The impression is that the Cool Jeans Company has sold almost three times as much as the Hip Jeans Company, and the Top Jeans Company has sold almost twice as much as the Hip Jeans Company.

c. The Cool Jeans Company would be the one most likely to present information in this manner.

3. a. You get the impression that interest rates are plummeting even though the change is not all that large.

b. A bank, the government, or any company that is trying to promote falling interest rates might present the data like this.

c. You can have the bars representing the rates begin on the same horizontal line. In addition, you can take out the arrow curving downwards.

4. a. No, the computer store may have lost money last year and just be breaking even this year. Also, operating costs may have increased significantly.

b. Consumers and people interested in buying stocks in that company would be affected.

5. a. No, the graph is about eye injuries in sports. You don't have enough information to make a conclusion on the overall dangers of each sport.

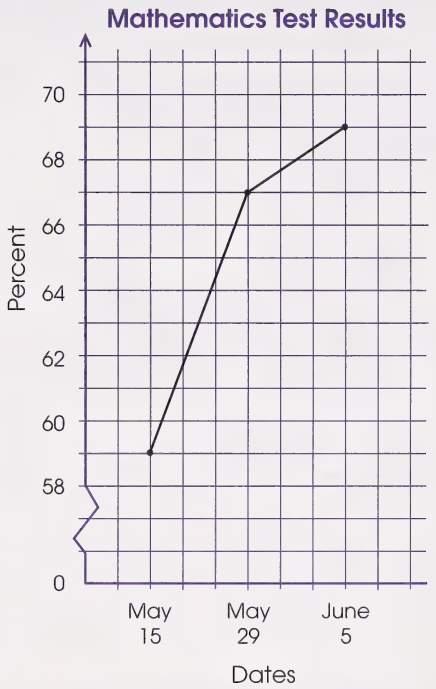
b. Yes, the information does show that hockey is a dangerous sport in terms of eye injuries.

c. You would need the number of participants in each sport as well as the kind and severity of every injury.

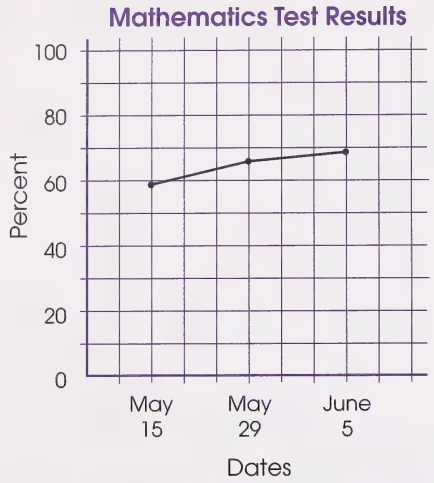
6. a.
- The claim doesn't tell you what it is being compared to.
- The claim seems to be saying that there is 30% less fat in their fries than in any competitors' fries.
- The claim might be saying that there is 30% less fat in their fries than there used to be.

b.
- The claim doesn't tell you how many more switched nor what others.
- The claim seems to be saying that more people started drinking their new kinds of soft drinks than started drinking any other new kind of soft drink.
- The claim might be saying that more switched from their old line of soft drinks to their new line of soft drinks than to any other line of soft drinks.

7. a.



b.



c. Your graph is used to show your improvement in as good a light as possible. The teacher's graph compares your progress over the whole scale of possible marks.

8. Answers will vary. Here are some possible reasons.

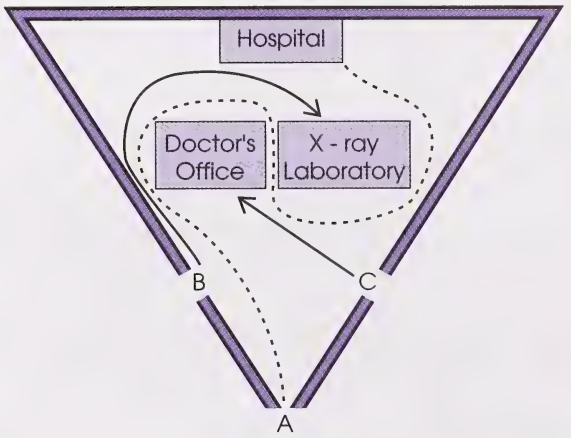
- for personal gain
- to help sell a product
- to cover up their errors or misfortunes
- to make themselves or their group look better than a competitor
- to influence people who are trying to make a decision

9. Answers will vary. Support your answer with reference to the article and show what changes you would make.

Now Try This

10. A ring that is 75% pure gold would have a rating of 18 karats $(24 \times 75\% = 18)$.

11.



Section 2: Follow-up Activities

Extra Help

1. a. Instead of using a picture to represent every 10 pizzas, different-sized pictures are used to represent the different frequencies.

The number of people who prefer pepperoni is 2.25 times the number of people who prefer anchovies. However, the picture representing the people who prefer pepperoni is 2.25 times as wide and 2.25 times as high; so, it appears much greater.

b. Because the scale on the vertical axis is broken, the height of the bars are out of proportion.

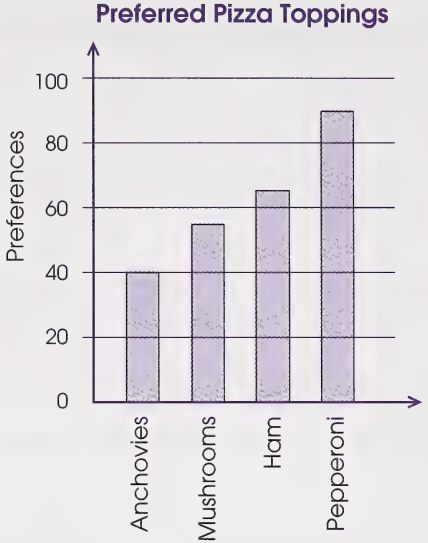
The height of the pepperoni bar should be 2.25 times the height of the anchovies bar. Instead, the height of the pepperoni bar is over 4 times the height of the anchovies bar.

2. a.

Preferred Pizza Toppings

Anchovies	
Mushrooms	
Ham	
Pepperoni	
Legend:	represents 10 preferences

b.



Enrichment

1. Answers will vary. You should describe what the graph contains and how it is creative in terms of the topic or issue with which it deals.

2. Answers will vary. Your discussion should show how the graph was used to mislead the reader.

Section 3: Activity 1

1. $P(1s) = \frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{36}$

$P(6s) = \frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{36}$

$P(1s \text{ and } 6s) = \frac{1}{36} \times \frac{1}{36}$

$= \frac{1}{1296}$

2. a. $P(3T) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{8}$

The probability of all three coins coming up tails is $\frac{1}{8}$.

b. The probability of getting heads on the dime is $\frac{1}{2}$.

The probability of getting tails on the nickel is $\frac{1}{2}$ and tails on the quarter is $\frac{1}{2}$.

Thus, the probability of all three events occurring is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

c. The probability for heads on the dime, tails on the nickel, and tails on the quarter is the same as the probability for three tails coming up. This indicates that the events are equally likely (the probability of each separate event is the same) to occur.

d. Other possible events are HHT, HTH, THH, THT, TTH, and HHH. The probability of each of these events occurring is also $\frac{1}{8}$. **Note:** The order of dime, nickel, and quarter is kept the same for each event.

3. a. Neither Lawrence nor Heather are correct. The next toss does not depend on the previous tosses.

b. The probability of heads on the next toss is $\frac{1}{2}$.

4. The probability of selecting each single digit is 1 out of 10 or $\frac{1}{10}$.

Therefore, the probability of someone guessing Sonia's combination is $\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{1000}$.

5. The probability of not getting a 3 or 4 on one die is $\frac{4}{6}$ or $\frac{2}{3}$. Thus, the probability of not getting a 3 or 4 on either die is $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$.

For every nine tosses you would move backward four times and forward five times. Thus, you would move $4 \times 2 = 8$ spaces backward and 5 spaces forward.

Therefore, after nine tosses you would probably have moved backward.

6. Use two dice. Toss the dice nine times and record the numbers that come up in a chart. Count the number of times a 3 or 4 did not come up on a toss.

Your chart may look similar to the following.

Toss of Dice	Number Other Than 3 or 4	3 or 4 Comes Up
1		✔
2		✔
3	✔	
4	✔	
5		✔
6	✔	
7		✔
8	✔	
9		✔

a. You move backward $4 \times 2 = 8$ spaces.
You move forward $5 \times 1 = 5$ spaces.
You are moving backward after 9 tosses.

b. A 3 or 4 came up five times after 9 tosses. This is exactly the theoretical probability that was calculated in the answer to question 5. You may not get the theoretical probability in your results.

7. a. The probability of drawing a purple candy is $\frac{3}{9}$ or $\frac{1}{3}$.

b. You would have to draw six candies before being certain of drawing a purple one (assuming no purple candies were drawn on the first six draws).

8. a. $P(3H) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{8}$

The probability of getting three heads is $\frac{1}{8}$.

b. Since there are three coins, there are three ways of getting a head and two tails (HTT, THT, and TTH).

The probability of each way is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

The probability of the three ways is $\frac{1}{8} + \frac{1}{8} + \frac{1}{8}$ or $\frac{3}{8}$.
Therefore, the probability of getting a head and two tails is $\frac{3}{8}$.

9. $P(\textit{rain each day}) = P(\textit{rain on Monday}) \times P(\textit{rain on Tuesday}) \times P(\textit{rain on Wednesday})$

$$= \frac{60}{100} \times \frac{50}{100} \times \frac{30}{100}$$
$$= 0.6 \times 0.5 \times 0.3$$
$$= 0.09 \text{ or } 9\%$$

The chance of rain on all three days is 9%.

10. **a.** $P(\textit{yes and yes}) = 0.70 \times 0.70$
$= 0.49 \text{ or } 49\%$

Mary has a 49% probability that she will make both shots.

b. $P(\textit{no and no}) = 0.30 \times 0.30$
$= 0.09 \text{ or } 9\%$

Mary has a 9% probability of missing both shots.

c. $P(\textit{yes and no}) = 0.70 \times 0.30$
$= 0.21 \text{ or } 21\%$

Mary has a 21% probability of making the first shot and missing the second shot.

11. Answers will vary. A sample chart is given.

Sum	Tally	Occurances
2	𝍸	3
3	𝍸 I	6
4	𝍸 𝍸 I	11
5	𝍸 𝍸	10
6	𝍸 𝍸 𝍸 I	16
7	𝍸 𝍸 II	12
8	𝍸 𝍸 IIII	14
9	𝍸 𝍸	10
10	𝍸 III	8
11	III	3
12	𝍸 II	7

12. **a.** $\frac{3}{100} = 0.03$ **b.** $\frac{16}{100} = 0.16$

c. $\frac{12}{100} = 0.12$ **d.** $\frac{8}{100} = 0.08$

13. **a.** The probability of a sum of 2 is $\frac{1}{36}$. There is only one combination (1 and 1) that gives you a sum of two.

b. The theoretical probability of a sum of 2 is $\frac{1}{36} \doteq 0.03$.

The experimental probability of a sum of 2 is $\frac{3}{100} = 0.03$.

Thus, the probabilities are about the same.

14. a. The sum of 7 should appear most often. There are 6 combinations that give you a sum of 7 (1 and 6, 6 and 1, 2 and 5, 5 and 2, 3 and 4, and 4 and 3); the highest number of combinations for any other sum is 5.

b. From the given experimental results, the sum of 6 appeared most often. In your experiment, the 7 may have appeared more often.

15. a. Either 2 or 12 will appear least often. The theoretical probability for either is $\frac{1}{36}$.

b. In the given data, the 2 appeared least often as predicted but the 12 appeared more often than would be predicted. Your data may have been different.

16. There are not enough tosses for each sum to give an experimental value close to the theoretical value.

17. At times you may think of a sum and it does come up. It may even come up twice in a row. Some people feel that certain numbers are lucky for them. That is, they come up more often. This may be the case over several tosses; but over hundreds of tosses for each sum, the theoretical probability will hold out.

18. Your chart should be similar to the one in the answer to question 11.

19. Answers will vary. The following are possible probabilities.

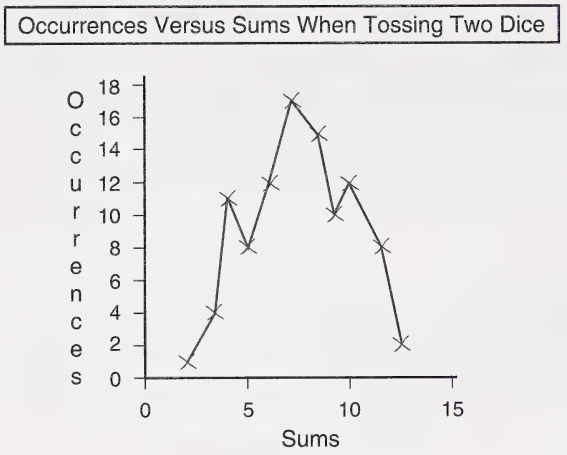
a. $\frac{4}{100} = 0.04$ **b.** $\frac{14}{100} = 0.14$

c. $\frac{16}{100} = 0.16$ **d.** $\frac{9}{100} = 0.09$

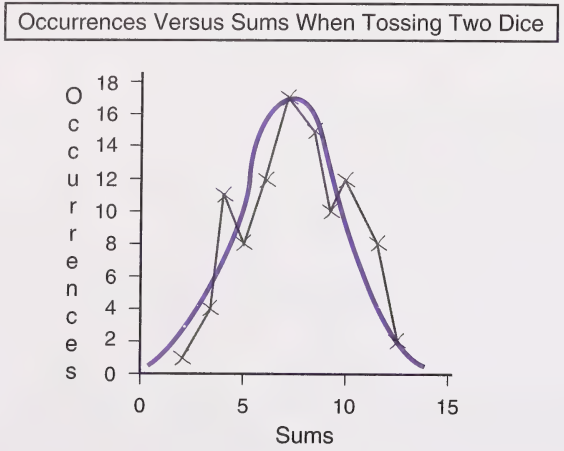
20. These results are similar to those in question 12.

21. The experimental probabilities should be getting closer to the calculated probabilities for each sum.

22. Your graph will look similar to the following.



23. Your smooth curve should look similar to the following. In statistics and probability, this is known as a bell-shaped curve.



24. **a.** 1, 1, 1 **b.** 1, 2, 2

c. 2, 1, 1 **d.** 2, 2, 2

Section 3: Activity 2

1. Answers will vary. You may have recorded responses like the following:

- I pick the date and last two digits of the year of birthdays and anniversaries of members of my family.
- I pick 2 or 3 of the numbers that came up last time and some numbers that have not come up for a long time.
- I play multiples of seven and eleven because they are lucky numbers.
- I use quick picks.
- I use the same numbers all the time.

2. Numbers that come up more often than others only appear to be doing so. Only a small part of the possible combinations are the numbers picked in a short time period (for example, 3 months). Over the long term, each number has the same probability of being drawn.

3. **a.** If you buy one ticket, the probability of winning the six-number prize is 1 in 13 983 816 or $\frac{1}{13\,983\,816}$.

b. If you buy ten tickets, the probability of winning the six-number prize are 10 in 13 983 816 or $\frac{10}{13\,983\,816}$ or $\frac{1}{1\,398\,381.6}$.

c. Yes, if you bought all the combinations you could be certain to win the six-number prize. However, there could be other winners so that you would have to share the jackpot.

4. Each person has a 1 in 700 000 probability of winning a share of the six-number prize.

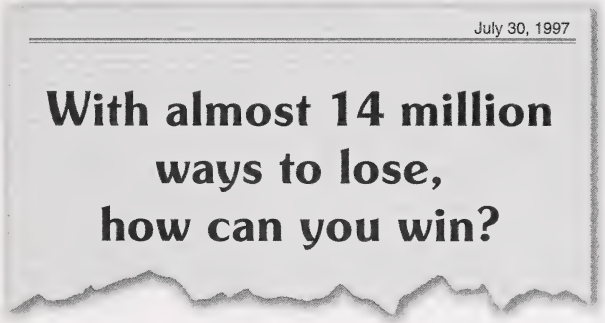
5. You would need about 1000 tickets to increase the probability to 1 in 14 000.

6. Although it may seem that some numbers have a higher probability of coming up than others, each six-number combination has the same probability of coming up.

7. There may be more than one winner because more than one person could buy the winning combination of numbers.

8. No, buying 57 tickets would not guarantee winning $10, but the probability would be quite high.

9. There is only 1 way to win in 14 million, not 14 million ways to win. To reflect the probability of winning it should read:



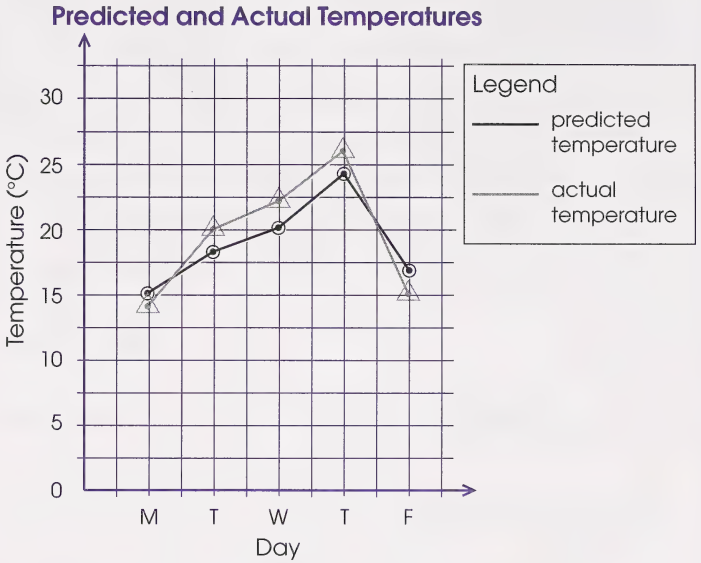
10. This is because the probability of winning any prize is a combination of the probabilities of winning any one prize. You have a greater probability of winning any prize than the probability of winning one particular prize.

11. **a.** A 60% chance of rain means that the probability of rain is 0.6. It also means given the predicted conditions, rain has fallen 60 times out of 100 in the past.

b. No, you cannot be certain that it will rain during some part of the weekend. It may rain part of the weekend, it may rain most of the weekend, it may rain in part of the forecast region; but not where René is camping.

c. He may look at the prediction of 60% rain and decide that there is a fairly good chance that it will rain. However, he may also recall that rain was predicted the last two weekends, and it didn't rain. He may then decide to go camping anyway; afterall, it may rain only part of the weekend.

12. **a.** Your graph may look something like the following.



The temperature increased more rapidly than predicted, but then cooled off more quickly as well.

b. Answers will vary. You may find that the actual and predicted amounts of precipitation may be very close or there may have been no rain when rain was predicted. In the case of the temperature predicted in question 12.a., the reason for the sudden drop in temperature on Friday was a cold front and a larger than predicted amount of precipitation.

13. The probability question the article is attempting to answer is, "What is the probability that a wallet, with $30 cash, lost on Edmonton streets will be returned?"

14. Twenty wallets were dropped on Edmonton streets. Thirteen wallets were returned with $30. One wallet was returned with $25, and one wallet was returned with no cash. Thirteen out of twenty wallets were returned with all the cash. Fifteen out of twenty wallets (or $\frac{3}{4}$ of the wallets) were returned.

15. The conclusion is that the probability is fairly high that your wallet would be returned if you lost it on a street in Edmonton. Three-quarters of the wallets were returned; so, you probably would agree with the conclusion.

16. Most of the wallets were dropped in the downtown area. This area would have most of the people walking about on the sidewalks.

17. No, there does not appear to be any bias. At first glance, the large number of wallets dropped off in the downtown area may suggest some bias, but this is where most of the people are out on the sidewalks during the day.

18. Yes, the conclusion can be made about the entire city since many people from the outer areas are in the downtown area during the day, and about $\frac{1}{3}\left(\frac{7}{20}\right)$ of the wallets were dropped in various areas of the city.

19. Answers will vary. You may not be able to answer all the questions for both examples.

Section 3: Follow-up Activities

Extra Help

1. The probability of spinning blue is $\frac{1}{4}$.

The probability of rolling a number less than 3 is $\frac{2}{6}$ or $\frac{1}{3}$.

The probability of both events occurring is $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$.

2. The probability of heads on a coin toss is $\frac{1}{2}$.

The probability of rolling an even number on a die is $\frac{3}{6}$ or $\frac{1}{2}$.

The probability of drawing an ace of spades from a deck of cards is $\frac{1}{52}$.

The probability of all three events occurring is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{52} = \frac{1}{208}$.

3. **a.** experimental result **b.** subjective judgement

c. theoretical calculation **d.** subjective judgement

Enrichment

1. a. An expected win of $4 is not a very good return on an investment of $10.

b. No. This is the amount you would get if everyone received an equal portion of the prize money.

c. People buy lottery tickets for various reasons. The following are some of these reasons.

- People hope to win the big prize.
- People feel they have a chance of winning some prize.
- People look at it as a donation. Lotteries in Canada cannot be run for individual profit. They must support some good cause such as a charity, a hospital foundation, or a community project.

2. a.
$$\frac{(1\times 3000)+(5\times 1000)+(10\times 100)}{10\,000} = \frac{3000+5000+1000}{10\,000}$$
$$= \frac{9000}{10\,000}$$
$$= 0.90$$

The expected winning is $0.90.

b.
$$\frac{(1\times 2500)+(1\times 1000)+(5\times 100)}{1500} = \frac{2500+1000+500}{1500}$$
$$= \frac{4000}{1500}$$
$$= 2.666\,666\,667$$

The expected winning is $2.67.

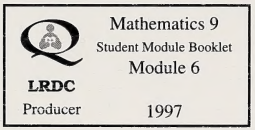
3. Question 2.a.

$$P(win) = \frac{16}{10\,000}$$
$$= 0.0016$$

Question 2.b.

$$P(win) = \frac{7}{1500}$$
$$\doteq 0.0047$$

The lottery in question 2.b. gives the better chance of winning a prize.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